A Gaussian Affine Term Structure Model of Interest Rates and Credit Spreads

Zhiping Zhou

Bocconi University

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Motivation

- A No-Arbitrage Finance Framework
  - The pricing kernel exists (prices all cash-flow)
- Ang and Piazzesi (2003)
  - Extend standard affine term structure models (ATSM) by incorporating two macro variables as priced risk factors
  - Identify their explanatory power for Treasury yields and ability to improve out-of-sample forecasts
- Targets: extend the paper of Ang and Piazzesi (2003) and price corporate bonds and Treasuries in a unified framework
  - Incorporate financial market variables (funding liquidity and market volatility)
  - Propose a minimum-chi-square method (MCSE) for estimation (Hamilton and Wu (2012, 2014))

Results Preview

- Macroeconomic factors display large explanatory power for Treasury yields and credit spreads
- Financial market factors have limited effects on the Treasury yield curve but substantial impacts on the credit spread
  - Positive volatility shocks increase credit spreads and this effect is stronger for low rated credit spreads
  - Liquidity factor has the strongest effects on short-term credit spreads among the four economic factors
  - The effects of macro variables are more persistent than the effects of financial market variables
- Forecasts for credit spreads improve when financial market factors are incorporated and when no-arbitrage restrictions are imposed.

Literature

- A growing literature that uses either reduced-form or structural models (Singleton (2006): Empirical Dynamic Asset Pricing)
  - These models summarize the variation on the term structure of interest rates and credit spreads
  - These studies are based on latent variables, directly derived from the yield curve and credit spread term structure
  - Only a small number of observable variables are incorporated for tractability reasons
- The economic meanings of these latent factors are unclear!

Literature

- A growing literature that jointly estimates the term structure of Treasury yields and corporate bond yields
  - Mueller (2008)
  - Wu and Zhang (2008)
- Gaussian ATSM: tremendous numerical challenges in estimating parameters due to highly non-linear likelyhood surfaces
  - Ang and Piazzesi (2003): a model with many factors
  - Mueller (2008): 2 billion sets of starting values
- We propose a minimum-chi-square method for the estimation
  - Uses linear regression to reduce the dimension of the numerical optimization problem
  - Eliminates parameters from the objective function that cause problems for MLE and QMLE

Literature

- The effects of aggregate liquidity or liquidity risk on asset pricing
  - Garleanu and Pedersen (2011): funding liquidity crisis can cause price gaps
  - Dick-Nielsen, Gyntelberg, and Lund (2013): funding liquidity drives the bond market liquidity in Denmark
- The relationship between illiquidity shocks and returns for corporate bonds
  - Acharya, Amihud, and Bharath (2013): impact of liquidity shocks on asset prices is stronger in adverse economic times
  - Lin, Wang and Wu (2011): liquidity risk determines corporate bond returns
- We study the effects of funding liquidity on corporate bond pricing in a no-arbitrage framework

The Model: Affine Functions

- The risk-free rate, r_t^f is assumed to be general affine functions of the underlying state vector:

$$r_t^f = \alpha_0 + \alpha_1 H_t,$$

where α_0 is a scalar, α_1 is an $N_h \times 1$ vector and H_t is an $N_h \times 1$ vector of state variables at time t.

- The stochastic discount factor (SDF):

$$M_{t+1} = \exp(-r_t^f - \frac{1}{2}x_t' x_t - x_t' \epsilon_{t+1}).$$

- The market price of risk, x_t, follow the affine specification:

$$x_t = \xi_0 + \xi_1 H_t,$$

in which ξ_0 is an $N_h \times 1$ vector and ξ_1 is an $N_h \times N_h$ matrix. Here the SDF is a quadratic function of x_t.

- The total gross return R_{t+1} of any nominal asset satisfies

$$E_t\left(M_{t+1} R_{t+1}\right) = 1.$$

The Model: State Variables

- The dynamics of $H_{t+1} = (H^{m'}_{t+1}\ H^{l'}_{t+1})'$ in compact form is given as a first order VAR:

$$H_{t+1} = \Theta_0 + \Theta_1 H_t + \Sigma_H \epsilon_{t+1},$$

where ϵ_{t+1} is an $N_h \times 1$ vector of independent standard normal shocks

- The risk-neutral investor believed that the factors are characterized by a Q-measure VAR

$$H_{t+1} = \Theta^Q_0 + \Theta^Q_1 H_t + \Sigma_H \epsilon^Q_{t+1},$$

where ϵ^Q_{t+1} is an $N_h \times 1$ vector of shocks under the Q-measure and

$$\begin{aligned} \Theta^Q_0 &= \Theta_0 - \Sigma_H \xi_0, \\ \Theta^Q_1 &= \Theta_1 - \Sigma_H \xi_1. \end{aligned}$$

The Model: Treasury Yields and Corporate Bond Yields

- Treasury bond prices are exponential affine functions of the state variables (Ang and Piazzesi (2003)):

$$P^{TB}(H_t, \tau) = \exp\{-A(\tau) - B(\tau)^T H_t\},$$

$$\begin{aligned} B(\tau+1) &= \alpha_1 + B(\tau)\Theta_1^Q, \\ A(\tau+1) &= \alpha_0 + A(\tau) + B(\tau)^T \Theta_0^Q - \frac{1}{2} B(\tau)^T \Sigma_H^T \Sigma_H B(\tau) \end{aligned}$$

- The defaultable bonds can be valued as if they were risk-free by replacing the short rate r_t^f with a default adjusted rate $r_t^f + s_t^i$,

$$s_t^i = \eta_0^i + \eta_1^i H_t + \epsilon_t^i,$$

- The time-t price of a zero-coupon defaultable bond for a certain credit-rating class i with time to maturity τ is

$$P_i^{CB}(H_t, \tau) = \exp(-A_i(\tau) - B_i(\tau)^T H_t),$$

$$\begin{aligned} B_i(\tau+1) &= \alpha_1 + \eta_1^i + B_i(\tau)\Theta_1^Q, \\ A_i(\tau+1) &= \alpha_0 + \eta_0^i + A_i(\tau) + B_i(\tau)^T \Theta_0^Q - \frac{1}{2} B_i(\tau)^T \Sigma_H^T \Sigma_H B_i(\tau) \end{aligned}$$

The Model: Corporate Bond Spreads

- The observed yields on zero coupon Treasury bonds are given by,

$$y(H_t,\tau) = -\frac{\log P^{TB}(H_t,\tau)}{\tau} = \frac{A(\tau)}{\tau} + \frac{B(\tau)^T}{\tau} H_t + e(t,\tau).$$

- The observed yields on zero coupon defaultable bond are given by,

$$y_i(H_t,\tau) = -\frac{\log P_i^{CB}(H_t,\tau)}{\tau} = \frac{A_i(\tau)}{\tau} + \frac{B_i(\tau)^T}{\tau} H_t + e_i(t,\tau).$$

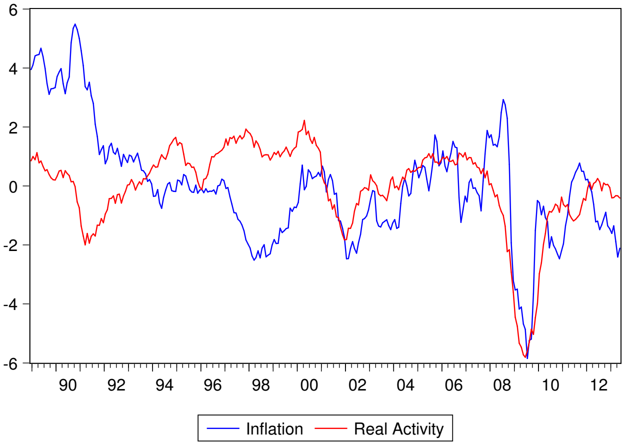
- Credit spreads can then be calculated as the difference between the yields on defaultable and default-free bonds:

$$\begin{aligned} CS_i(t,\tau) &= y_i(t,\tau) - y(t,\tau) \\ &= \left[\frac{A_i(\tau) - A(\tau)}{\tau}\right] + \left[\frac{B_i(\tau) - B(\tau)}{\tau}\right]^T H_t \\ &\quad + e_i(t,\tau) - e(t,\tau). \end{aligned}$$

Data

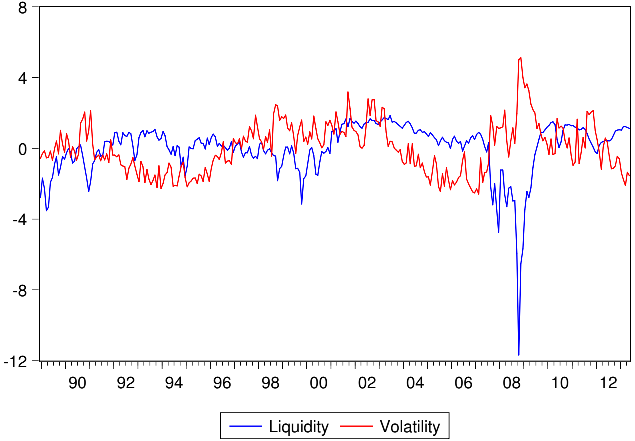
- Government bonds
  - 1m and 3m-Treasury yields from Fama CRSP Treasury Bill files
  - 24m, 60m and 120m-Treasury yields from Guerkaynak, Sack, and Wright data
- Corporate bonds
  - A and BBB-rated corporate yields from Merrill Lynch
  - Sample period: 1988:12 to 2013:05
- Economic variables: Extract 1st PCA
  - Inflation: PPI, CPI and PCEde
  - Real activity: GEMP, GIP and UE
  - Liquidity: 3m-TED, 6m-TED and 3-month CPMFFR
  - Volatility: VXO and VIX

Data: Inflation and Real activity



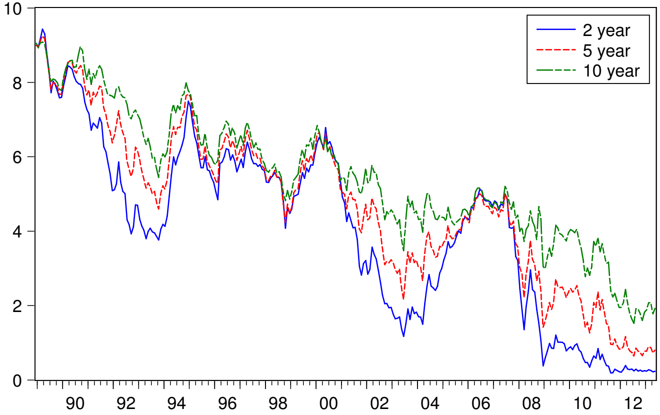
- Inflation factor peaks in early 1991 and then goes down
- Real activity factor moves at business cycle frequencies

Data: Liquidity and Volatility



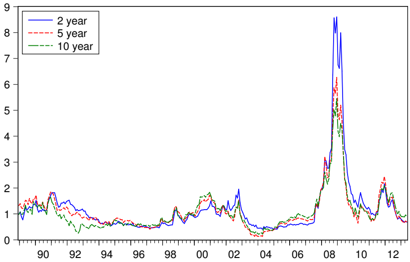
- Spikes in volatility and troughs in liquidity appear in financial market turbulence

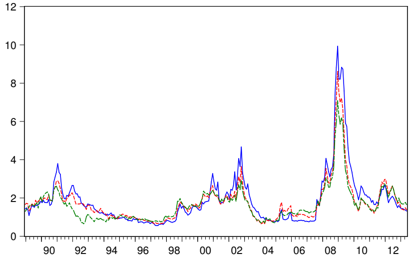
Data: Treasury Yields



- The movement in the Treasury yields is related to macro factors

Data: Credit Spreads





Econometric Methodology

- We adopt a VAR(1) for the underlying states:

$$\begin{aligned} H^m_{t+1} &= \Theta_{0m} + \Theta_{mm} H^m_t + \Theta_{ml} H^l_t + \Sigma_{mm} \epsilon^m_{t+1} \\ H^l_{t+1} &= \Theta_{0l} + \Theta_{lm} H^m_t + \Theta_{ll} H^l_t + \Sigma_{lm} \epsilon^m_{t+1} + \Sigma_{ll} \epsilon^l_{t+1} \end{aligned}$$

- (MLE) The structure model is given as

$$\begin{aligned} H^m_t &= \Theta_{mm} H^m_{t-1} + \Sigma_{mm} \epsilon^m_t \\ Y^1_t &= C_1 + D_{1m} H^m_t + D_{1l} H^l_t \\ Y^2_t &= C_2 + D_{2m} H^m_t + D_{2l} H^l_t + \Sigma_e u^e_t \end{aligned}$$

- (MCSE) The reduced-form model is given as

$$\begin{aligned} H^m_t &= \phi^*_{mm} H^m_{t-1} + u^*_{mt} \\ Y^1_t &= C^*_1 + \phi^*_{1m} H^m_{t-1} + \phi^*_{11} Y^1_{t-1} + \psi^*_{1m} H^m_t + u^*_{1t} \\ Y^2_t &= C^*_2 + \phi^*_{2m} H^m_t + \phi^*_{21} Y^1_t + u^*_{2t} \end{aligned}$$

Econometric Methodology

Table: Mapping between structural and reduced-form parameters

VAR parameters	No. of elements	Σ_e 5	Σ_{mm} 10	Θ_{mm} 16	$\xi_{1m,1l}$ 16+9	α_1 7	Θ_{ll} 6	η_0 1	η_1 7	α_0 1	ξ_0 7
Ω_2^*	5	×									
Ω_m^*	10		×								
ϕ_{mm}^*	16			×							
ψ_{1m}^*	12			×	×	×			×		
ϕ_{21}^*	15				×	×	×		×		
Ω_1^*	6				×	×	×		×		
ϕ_{11}^*	9				×	×	×		×		
ϕ_{2m}^*	20			×	×	×	×		×		
ϕ_{1m}^*	12			×	×	×	×		×		
C_2^*	5		×	×	×	×	×	×	×	×	×
C_1^*	3		×	×	×	×	×	×	×	×	×

Minimum-Chi-Square Estimation

- (MLE) We assume that $\widehat{R}$ is a consistent estimate of the information matrix, which satisfies

$$R = -\frac{1}{T}E\left[\frac{\partial^2 \mathcal{L}(\pi; Y)}{\partial\pi\partial\pi'}\right].$$

- (MCSE) The Wald statistic is calculated as

$$T\left[\widehat{\pi} - g(\theta)\right]' \widehat{R}\left[\widehat{\pi} - g(\theta)\right],$$

and its asymptotic distribution under the null hypothesis follows $\chi^2(q)$, where the degree of freedom q is the dimension of π.

- $\widehat{\theta}_{MCSE}$ are asymptotically equivalent to $\widehat{\theta}_{MLE}$

Results: Unrestricted Estimation for Treasury Yields

Maturity	2 year	t-Statistic	5 year	t-Statistic	10 year	t-Statistic
Intercept	4.101	(40.733)	4.735	(50.020)	5.458	(65.536)
Inflation	0.599	(11.085)	0.571	(11.241)	0.511	(11.434)
Real Activity	0.636	(8.806)	0.403	(5.926)	0.183	(3.066)
Adjusted R^2	0.480		0.418		0.366	
Intercept	4.101	(41.501)	4.735	(50.238)	5.458	(65.509)
Inflation	0.532	(9.254)	0.531	(9.692)	0.490	(10.117)
Real Activity	0.655	(8.764)	0.408	(5.730)	0.188	(2.991)
Liquidity	-0.273	(-3.521)	-0.156	(-2.114)	-0.085	(-1.297)
Volatility	-0.068	(-0.807)	-0.055	(-0.680)	-0.023	(-0.323)
Adjusted R^2	0.499		0.423		0.366	

- Inflation and real activity are significant in all regressions
- Adding financial market variables marginally improves Adj. R^2

Results: Unrestricted Estimation for A Credit Spreads

Maturity	2 year	t-Statistic	5 year	t-Statistic	10 year	t-Statistic
Intercept	1.220	(23.438)	1.129	(27.203)	1.074	(28.453)
Inflation	0.029	(1.044)	0.077	(3.434)	0.029	(1.417)
Real activity	-0.569	(-15.237)	-0.358	(-12.003)	-0.298	(-11.016)
Adjusted R^2	0.450		0.327		0.294	
Intercept	1.220	(42.634)	1.129	(48.559)	1.074	(48.310)
Inflation	-0.061	(-3.681)	0.011	(0.814)	-0.024	(-1.862)
Real activity	-0.463	(-21.392)	-0.268	(-15.221)	-0.214	(-12.715)
Liquidity	-0.461	(-20.525)	-0.349	(-19.117)	-0.295	(-16.918)
Volatility	0.128	(5.250)	0.127	(6.427)	0.135	(7.145)
Adjusted R^2	0.834		0.789		0.755	

- All coefficients on liquidity and volatility are significant
- Adding financial market variables improves Adj. R^2

Results: Unrestricted Estimation for BBB Credit Spreads

Maturity	2 year	t-Statistic	5 year	t-Statistic	10 year	t-Statistic
Intercept	1.871	(35.221)	1.750	(37.138)	1.642	(38.393)
Inflation	0.002	(0.082)	0.034	(1.340)	0.010	(0.445)
Real activity	-0.765	(-20.057)	-0.527	(-15.588)	-0.409	(-13.311)
Adjusted R^2	0.595		0.460		0.388	
Intercept	1.871	(52.842)	1.750	(53.870)	1.642	(57.735)
Inflation	-0.039	(-1.878)	-0.009	(-0.487)	-0.022	(-1.349)
Real activity	-0.634	(-23.678)	-0.420	(-17.084)	-0.303	(-14.077)
Liquidity	-0.305	(-10.958)	-0.286	(-11.194)	-0.244	(-10.930)
Volatility	0.280	(9.267)	0.214	(7.715)	0.228	(9.384)
Adjusted R^2	0.820		0.743		0.729	

- Results are similar to those for A credit spreads
- Financial market factors $\rightarrow$ the dynamics of credit spreads

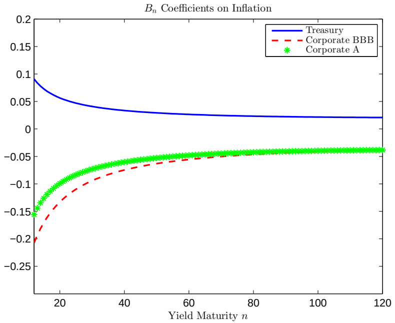
Results: Estimation with No-Arbitrage Restrictions

Θ_{1l}	0.9386	0	0	Θ^{Q}_{1l}	0.6660	0	0
	(76.934)				(34.426)		
	0	0.9934	0		-0.2778	0.9934	-0.0255
		(764.15)			(-10.289)	(764.15)	(-6.8919)
	0	0.0016	0.8262		-0.0298	0.0016	0.9793
		(2.2857)	(42.588)		(-2.0694)	(2.2857)	(489.65)
α_0	0.0027						
	(27.000)						
α_1	0.0004	0.0006	-0.0002	4.94E-06	1.46E-04	1.71E-04	6.51E-05
	(8.8300)	(10.187)	(-3.2733)	(0.0744)	(12.586)	(21.242)	(5.9724)
η_0^{BBB}	0.0044						
	(42.718)						
η_1^{BBB}	-0.0007	-0.0002	-0.0008	0.0002	3.72E-04	1.84E-04	-1.75E-04
	(-0.7928)	(-0.2488)	(-9.0293)	(3.3113)	(4.9799)	(16.140)	(-11.218)
η_0^{A}	0.0036						
	(32.143)						
η_1^{A}	-0.0011	-0.0008	-0.0020	-0.0008	-6.73E-05	1.48E-04	-9.08E-05
	(-1.4056)	(-2.162)	(-20.243)	(-5.2980)	(-2.6289)	(11.128)	(-11.792)

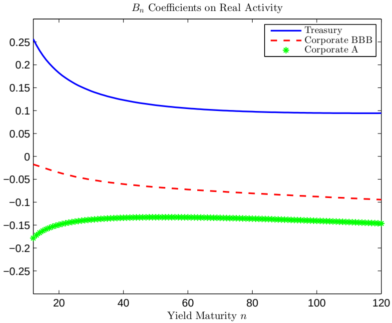
Results: Estimation with No-Arbitrage Restrictions

- Latent factors are highly persistent
- The average compensation for short-term credit risk is higher for higher-rated bonds
- Financial market factors are more important for credit spreads than for Treasury yields
- Liquidity factor is more important than the volatility factor for capturing the dynamics of credit spreads

Results: Factor Loadings on Macro Factors



(a) Inflation factor

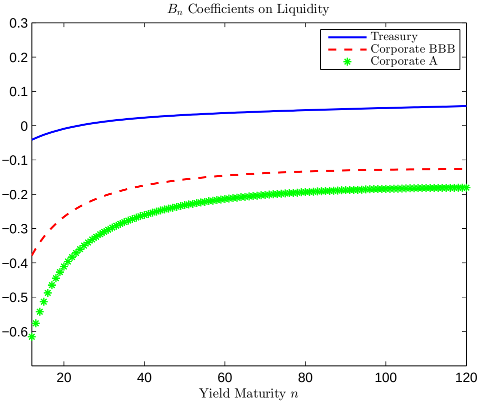


(b) Real activity factor

- Positive for Treasury yields and negative for corporate yields
- Larger effects on short yields than on long yields

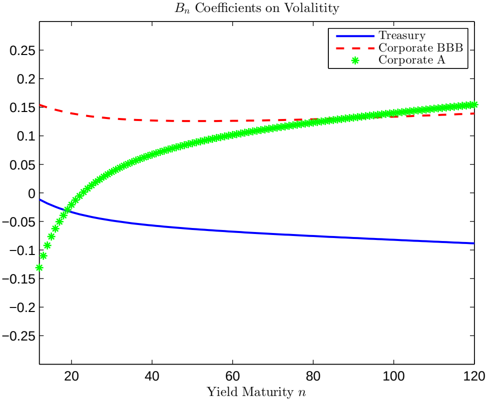
Results: Factor Loadings on Funding Liquidity Measure

IRF



- The loadings on liquidity for corporate yields are negative
- Initial reaction to liquidity shocks is stronger for A-rated bond

Results: Factor Loadings on Market Volatility Measure



- The loadings on volatility for BBB corporate yields are positive
- Short and long yields (A-rated) react to the volatility change in different directions

Forecasts Comparisons

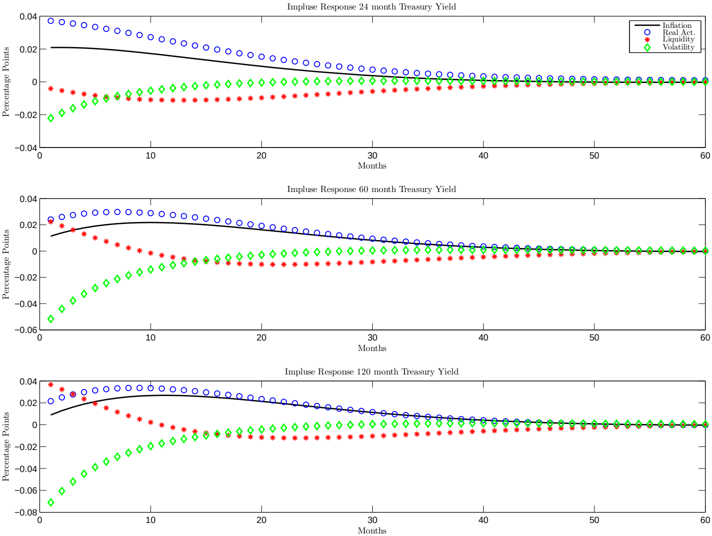
	RMSE criteria			MAD criteria		
	Two Factors	Four Factors	No-Arbitrage	Two Factors	Four Factors	No-Arbitrage
A24	0.0390	**0.0192**	0.0246	0.0268	**0.0126**	0.0146
A60	0.0380	0.0357	**0.0277**	0.0321	**0.0198**	0.0216
A120	0.0357	0.0367	**0.0312**	0.0302	**0.0191**	0.0223
BBB24	0.0442	0.0263	**0.0211**	0.0276	0.0215	**0.0117**
BBB60	0.0492	0.0411	**0.0410**	0.0430	**0.0307**	0.0317
BBB120	0.0462	0.0458	**0.0453**	0.0358	**0.0298**	0.0303

- Incorporating financial market variables improves forecasts
- Imposing no-arbitrage restrictions improves forecasts
- RMSE penalizes large errors more than MAD

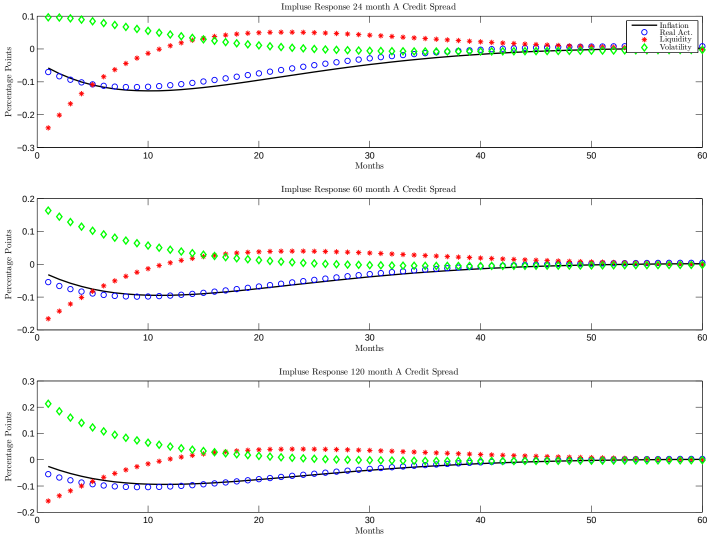
Concluding Remarks

- Conclusions:
  - Macro factors are important determinants of both Treasury yields and credit spreads
  - Financial market factors only have marginal effects on the Treasury yield curve but exert substantial impacts on the credit spread term structure
  - Adding financial market factors and imposing no-arbitrage restrictions help in forecasting
- Policy Implications: Funding Liquidity and Volatility
- Future Research:
  - Apply this framework to other assets, such as equity
  - Extend this framework by incorporating regime switches

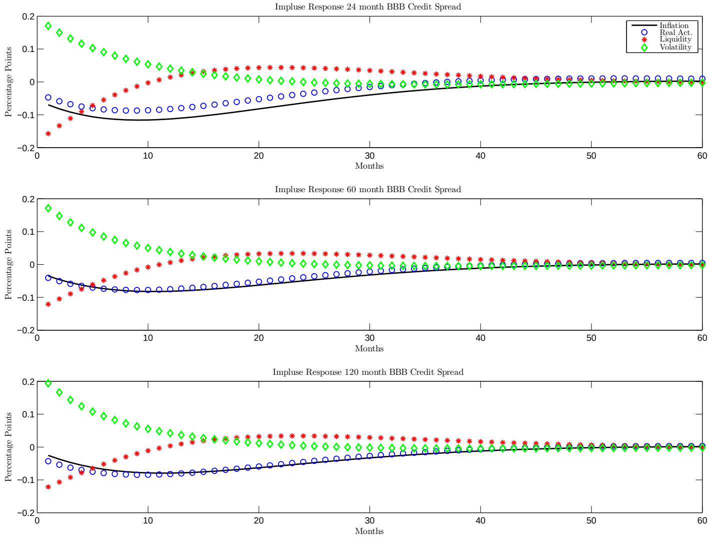
Impulse Responses for Treasury Yields



Impulse Responses for A Credit Spreads



Impulse Responses for BBB Credit Spreads



Impulse Responses

Factors

- The impulse responses for all factors are large in absolute value at short horizon and level off slowly towards zero.
- Macroeconomic shocks have more persistent impacts on both Treasury yields and credit spreads than financial market shocks.