A Gaussian Affine Term Structure Model of Interest Rates and Credit Spreads

Zhiping Zhou[*]

November 3, 2014

Abstract

We estimate a no-arbitrage term structure model of U.S. Treasury yields and corporate bond spreads with both economic factors and latent factors as drivers of term structure dynamics. We consider two sets of economic factors: macro factors consisting of inflation and real activity, and financial market factors consisting of funding liquidity and market volatility. We show that financial market factors have limited effects on the Treasury yield curve but substantial impacts on the credit spread term structure. In particular, negative liquidity shocks widen credit spreads, and this effect is more pronounced for short-term corporate bonds. We also find that out-of-sample forecasts for credit spreads improve when financial market factors are incorporated and when no-arbitrage restrictions are imposed. We also propose a minimum-chi-square method for estimating the term structure models of interest rate and credit spreads, which is more efficient and accurate than the widespread maximum-likelihood estimation.

Keywords: Term structure; Yield curve; Credit spreads; Affine model

JEL Codes: E43; C13; G12; G13

---

[*]Department of Finance, Bocconi University, Milan 20142, Email: zhiping.zhou@phd.unibocconi.it. Preliminary and Please DO NOT Quote.

1 Introduction

In this study, we propose a novel framework to identify what drives the relationship between the dynamics of economic factors, including macroeconomic factors and financial market factors, and the term structure of interest rates and credit spreads. Numerous empirical studies show that credit spreads depend crucially on the state of the macroeconomy and the state of the financial market. A partial list of studies along this line of research includes Carey (1998), Collin-Dufresne, Goldstein, and Martin (2001), Elton, Gruber, Agrawal, and Mann (2001), Altman, Brady, Resti, and Sironi (2005), Longstaff, Mithal, and Neis (2005), Giesecke, Longstaff, Schaefer, and Strebulaev (2011), Lin, Wang, and Wu (2011), Acharya, Amihud, and Bharath (2010), and Giesecke, Longstaff, Schaefer, and Strebulaev (2011). Though rich in economic intuitions, the results of these studies depend on the specific choices of the explanatory variables and the characteristics of credit spreads, such as maturity and credit-rating. Previous literature has mainly focused on explaining changes in credit spreads using regression analysis.

This paper, by contrast, investigates credit spreads in a multi-factor (macro factors and financial market factors) term structure model subject to restrictions imposed by no-arbitrage assumptions. Using a no-arbitrage framework not only facilitates the interpretation of the estimation results, but it also provides insights into how yields of maturities not included in the regressions move. In our setup, defaultable corporate bonds are valued as if they were risk-free by replacing the short rate with a default adjusted rate, where the default-adjusted rate for a certain credit-rating class is an affine function of the underlying factors (Duffie and Singleton (1999) and Duffie, Pedersen, and Singleton (2003)). One innovation of our framework is that the underlying factors are comprised of both observable economic variables—macroeconomic variables and financial market variables—and unobserved latent factors. This paper seeks to extend previous studies by drawing additional insights from the inclusion of macroeconomic variables, which consist of inflation and real activity and financial market variables, such as funding liquidity and market volatility.

2

Our main results are as follows. Macroeconomic factors display large explanatory power for Treasury yields. Positive shocks to both inflation and real activity increase Treasury yields, and these effects are stronger for short-term Treasury bonds. In addition, macroeconomic factors are also important determinants of credit spreads. Declines in inflation and real activity lead to higher credit spreads. In contrast, financial market factors have limited effects on the Treasury yield curve but substantial impacts on the credit spread term structure. Positive volatility shocks increase the BBB credit spreads, and this effect is stronger at the long end of the credit spreads curve. Their impacts on the A credit spreads are different over the whole credit spreads term structure, as they are negative at the short end but become positive in the middle. More importantly, liquidity factor has the strongest effects on the short-term credit spreads among the four economic factors. Funding liquidity shrinkage remarkably widens credit spreads, providing an explanation for the run-up in credit spreads in the recent global financial crisis. The impulse response functions show that the effects of macroeconomic factors on both Treasury yields and credit spreads are generally more persistent than the effects of financial market factors. Moreover, to investigate the role of financial market factors and no-arbitrage assumptions in forecasting credit spreads, we perform a comparison of out-of-sample forecasts for credit spreads. It turns out that adding financial market factors substantially improves forecasts for credit spreads. This improvement in forecasting performance is more noticeable for short-term credit spreads. Imposing no-arbitrage restrictions further helps in out-of-sample forecasts.

This work builds on recent studies of affine term structure models of default-free bonds with macroeconomic variables. In a seminal work, Ang and Piazzesi (2003) have introduced two macro factors, inflation and economic growth, as priced risk factors in a Gaussian affine term structure model of U.S. Treasury yields by using a factor representation of the pricing kernel. Inspired by their work, a rich literature has emerged that explores the importance of incorporating macro factors into no-arbitrage term structure models (see, for example, Gallmeyer, Hollifield, and Zin (2005), Diebold, Rudebusch, and Boragan Aruoba (2006), Hördahl, Tristani, and Vestin (2006), Wachter (2006), Bekaert, Cho, and Moreno (2010), Bikbov and Cher-

nov (2010)). However, little has been done to expand this literature beyond the pricing of Treasury bonds. As a natural extension to Ang and Piazzesi (2003), this paper incorporates corporate bonds into analysis and aims at pricing both default-free Treasury bonds and default-able corporate bonds in a uniformed no-arbitrage framework with macroeconomic variables and financial market variables as drivers of term structure dynamics.

A large strand of literature uses either reduced-form or structural models to describe the term structure of interest rates and credit spreads. Prominent examples include Longstaff, Mithal, and Neis (2005), Longstaff and Schwartz (1995), Duffee (1999), Collin-Dufresne, Goldstein, and Martin (2001), Eom, Helwege, and Huang (2004), Driessen (2005), Collin-Dufresne, Goldstein, and Helwege (2010) and Joslin, Singleton, and Zhu (2011). However, most of these studies are solely based on latent factors, which are directly derived from the yield curve and credit spread term structure. The economic meanings of these latent factors are unclear. Some studies consider observable variables, but the number of these observable variables is small due to tractability reasons. In contrast, we use two macro factors, inflation and real activity, and two financial market factors, funding liquidity and market volatility, to summarize the information and suppress the noises in many observable macroeconomic and financial times series.

There is a growing literature that jointly estimates the term structure of Treasury yields and corporate yields in linear affine models with macroeconomic variables, including Luisi and Amato (2006), Mueller (2008), and Wu and Zhang (2008). Luisi and Amato (2006) estimate a version that combines macroeconomic variables and latent factors. By using a macro-finance term structure model, Mueller (2008) tries to capture the joint dynamics of GDP, inflation, Treasury yields, and credit spreads. Another important paper in this research area is Wu and Zhang (2008), who links the dynamics and market pricing of three risk dimensions (inflation, real output growth, and financial market volatility) to the term structure of U.S. Treasury yields and corporate bond credit spreads. Instead of identifying three risk dimensions, we go one step further in identifying and estimating risk dimensions using macroeconomic factors, financial market factors, and the traditional latent factors in a novel

Gaussian affine term structure framework. This approach uses linear regression to reduce the dimension of the numerical optimization problem, and it improves the numerical behavior of estimation by eliminating parameters from the objective function that cause problems for conventional maximum-likelihood estimation (MLE) and quasi-maximum likelihood estimate (QMLE) methods.

Gaussian affine term structure models (ATSM) have become fundamental tools for empirical research in macroeconomics and finance. However, tremendous numerical challenges emerge in estimating linear affine models using conventional MLE due to highly non-linear and badly behaved likelihood surfaces. Ang and Piazzesi (2003) find difficulties in estimating models with many factors by using the MLE and try to achieve the global maximum by using multiple starting values. Their results were recently criticized by Hamilton and Wu (2012), who show that Ang and Piazzesi's parameter estimates in fact correspond to a local maximum of the likelihood surface. Mueller (2008) evaluates the likelihood for two billion sets of starting values and then optimizes them by using the best twenty thousand points as starting values. The number of the starting points itself demonstrates that the methodology used in this framework is not reliable. Luisi and Amato (2006) estimate the parameters using the MLE, a method similar to that used by Ang and Piazzesi (2003). Thus, previous studies estimating affine term structure models of Treasury yields and corporate yields suffer the same problem as Ang and Piazzesi (2003). Following Hamilton and Wu (2012, 2014), we develop a minimum-chi-square estimation (MCSE) method for estimating the term structure models of interest rate and credit spreads, which turns out to be much more efficient and accurate than the MLE.

The effects of aggregate liquidity or liquidity risk on asset pricing have been recently highlighted in the literature. Brunnermeier and Pedersen (2009) provide a model that elaborates on the relationship between funding liquidity and market liquidity (FL-ML) and show that the two notions are mutually reinforcing, leading to liquidity spirals. Garleanu and Pedersen (2011) show that a funding liquidity crisis gives rise to a price gap between securities with identical cash flows but different margins. Dick-Nielsen, Gyntelberg, and Lund (2013) find

5

that the ease of obtaining term funding in the money markets determines the liquidity in the bond market. The relationship between illiquidity shocks and returns of corporate bonds have been documented by De Jong and Driessen (2012), Acharya, Amihud, and Bharath (2010), and Lin, Wang, and Wu (2011). Acharya, Amihud, and Bharath (2010) show that the impact of liquidity shocks on asset prices is conditional and is significantly stronger in adverse economic times. Lin, Wang, and Wu (2011) establish that liquidity risk is a key determinant of corporate bond returns. Given the evidence that funding liquidity is positively correlated with corporate bond market liquidity, we adopt it in our framework and study its effects on corporate bond pricing in a no-arbitrage framework.

The rest of the paper is organized as follows. Section 2 describes the data. Section 3 specifies the no-arbitrage term structure model. Section 4 discusses the estimation and identification strategy. Section 5 presents the estimation results both with and without arbitrage restrictions and discusses the implied impulse responses and forecasting results. Section 6 concludes.

2   Data

This section provides a detailed description of the data used in this study. Treasury yields and corporate bond yields are the observable data. The starting point of the sample period is determined by the first month in which the corporate bond yields are available.

2.1   Economic Variables

We consider four groups of economic variables: inflation-related series, real activity-related series, volatility-related series, and liquidity-related series. In the group of inflation-related series, we have five inflation measures: the consumer price index (CPI), the core CPI, the personal consumption expenditure (PCE) deflator, the core PCE deflator, and the producer price index (PPI). The CPI measures the average changes in the price level of a basket of goods

6

and services bought by a typical household. The PCE deflator measures the average changes in the price level of a basket of goods and services bought by a typical consumer. The core measures of CPI and PCE exclude food and energy, because their prices are highly volatile. The PPI measures average changes in prices received by domestic producers for their output. The second group consists of three output and employment series. UE (Unemployment Rate) measures the percentage of people who are without work and are actively seeking work compared to all individuals in the labor force. GEMP (Growth Rate of Employment) measures the growth rate of the percentage of remaining employed people in the labor force. GIP (Growth Rate of Industrial Production) measures the growth rate of the production of goods. Overall, the first two groups of variables are used to characterize the economy as a whole and are called "macro variables."

In addition to macro variables, we also consider the economic variables that measure the dynamics of the financial market, which are called "financial market variables". On the one hand, to capture the liquidity level of the financial market, we adopt the TED spread as a proxy for the level of funding liquidity, which is defined as the difference between the 3-month LIBOR and the 3-month U.S. Treasury bill rate.[1] This measure has been suggested by Hameed, Kang, and Viswanathan (2010), Boyson, Stahel, and Stulz (2010), and Brunnermeier and Pedersen (2009). Other liquidity measures are 6MTED (6-month LIBOR minus T-bill spread) and 3MCPMFFR (3-month commercial paper minus federal funds rate). Following Wu and Zhang (2008), we incorporate the dynamics of the financial market volatility. Specifically, we use two volatility indexes from option market: the VIX index and the VXO index measure the one-month at-the-money Black and Scholes (1973) implied volatility on the S&P 500 index and the S&P 100 index, respectively. They take the yearly moving average of the daily volatility series and use the last day of each month to obtain their monthly data. Both series are available daily from the Chicago Board of Options Exchange, but the VIX series starts on a later date in January 1990. We augment the VIX data with the estimated data from December 1988 to December 1989.

---

[1]An alternative proxy for funding liquidity in this paper is the LIBOR-OIS spread. The results based on this alternative spread are quantitatively similar and are available upon request.

Following Ang and Piazzesi (2003) and Wu and Zhang (2008), to reduce the dimensionality of the system, we first normalize each series separately to have zero mean and unit variance and then extract the first principal component of each group of series separately. This leaves us with four economic variables, which are referred to as "inflation," "real activity," "volatility," and "liquidity." In what follows, we use the inflation and real activity factors to capture the information about the macroeconomic economy and volatility and liquidity factors to capture the information about the financial market.

Figure 1 plots the inflation and real activity factors. The inflation factor peaks in early 1991, which corresponds to the inflation pressure generated by oil shocks during the first Gulf War. In the subsequent period, the inflation factor shows a downward trend and stays low. As expected, most of the movement in the real activity factor is displayed at business cycle frequencies. For example, the real activity factor steadily grows in the period before 2008, but slumps at the onset of the recent global financial crisis. Figure 2 plots the financial market liquidity and volatility factors. The volatility factor has a couple of spikes over the sample period in response to financial market turbulence, such as the Asian crisis in 1997, the dot-com bubble burst in 2001, and the subprime mortgage crisis in 2008. Similarly, the liquidity factor fluctuates considerably over the sample period. Its troughs coincide with the spikes of the volatility factor, which together characterize major financial crises in the sample period. Table 1 shows the summary statistics of the four groups of economic factors that we consider. An important stylized fact is that all series are highly autocorrelated. This is consistent with the slow mean reversion observed in Figure 1 and Figure 2.

2.2 Government Bonds

The Treasury yields data are monthly continuously compounded spot rates at maturities 1 and 3 months and 2, 5, and 10 years from December 1988 to May 2013. The starting point of the sample period is determined by the availability of the corporate yields data, which will be discussed below. The sources of the data are twofold. The long maturity rates of 2, 5, and

10 years are from the dataset provided by Guerkaynak, Sack, and Wright,[2] while the short maturity rates of 1 and 3 months are from the Fama CRSP Treasury Bill files. The reason that we use the latter data is that the shortest yield maturity in the Guerkaynak, Sack, and Wright's dataset is one year and they advise against using their estimated yield curve parameters to calculate arbitrary maturity rates.

Figure 3 plots the time series of the Treasury yields at three maturities: 2, 5, and 10 years. The three long-term rates exhibit a downward trend over the sample period, largely matching the downward trend of the inflation factor observed in Figure 1. There are three periods of low short-term interest rates and high-term spreads around 1993, 2003, and 2008, each of which corresponds to a trough of the real activity factor in Figure 1. These comparisons suggest that the movement in the Treasury yields is related to macro factors.

2.3   Corporate Bonds

Following Wu and Zhang (2008), we construct continuously compounded spot rates for corporate bonds in the A and BBB rating classes, using month-end prices on corporate bonds that are either in the Merrill Lynch U.S. High Yield Index or the Merrill Lynch U.S. Corporate Master Index. The Merrill Lynch data covers the period from January 1997 to May 2013. To construct a longer sample, we augment the Merrill Lynch data by the Lehman Brothers Fixed Income database from December 1988 to December 1996. Therefore, we estimate our model based on data from December 1989 to May 2013.

To construct the spot rates, we estimate the Nelson and Siegel (1987) model on month-end prices of corporate bonds. The estimation chooses senior unsecured bonds with price quotes, with a fixed coupon schedule, and with maturities between 1 and 35 years, but without option features. Then, we calculate credit spread at three selected maturities, that is, 2, 5 and 10 years, for the A and BBB rating classes as the difference between the spot rate of the rating

---

[2]Gurkaynak, Sack, and Wright (2007) estimate the Treasury yield curve using a simple smoothing method and make their data publicly available on the Federal Reserve Board website (last update: December 22, 2011, http://www.federalreserve.gov/pubs/feds/2008/200805/200805abs.html).

class and the Treasury yield with the same maturity.

Figure 4 plots lots the time series of credit spreads in percentage at three maturities (2, 5, and 10 years) for the A and BBB rating classes. The credit spreads are higher for A rating class than for BBB rating class. Despite of the magnitude difference, the time series of different credit spreads exhibit strong common movements. We observe three common periods of high spreads, corresponding to the three recessions in the sample period. In particular, the spikes around 2008 highlight the periods of the highest spreads that occurred in the recent global financial crisis.

Table 2 reports the summary statistics of Treasury yields and credit spreads. There are some noteworthy stylized facts that characterize our bond yields data. The average Treasury yield curve is upward sloping, while the average credit spread curve is downward sloping; all series are highly autocorrelated. For a certain maturity, credit spread is higher for the lower rating class.

3   A No-Arbitrage Term Structure Model

3.1   State Variables

Let H_t be an $N_h \times 1$ vector of state variables at time t and ϵ_{t+1} be an $N_h \times 1$ vector of independent standard normal shocks. The observable vector H_t^m contains macroeconomic variables and financial market variables, while H_t^l only contains latent factors. We write the dynamics of H_{t+1} as a first order VAR:

$$H_{t+1} = \Theta_0 + \Theta_1 H_t + \Sigma_H \epsilon_{t+1}, \tag{1}$$

where Θ_0 is $N_h \times 1$, Θ_1 is $N_h \times N_h$, and Σ_H is $N_h \times N_h$. The risk-free rate, r_t^f is assumed to be general affine functions of the underlying state vector:

$$r_t^f = \alpha_0 + \alpha_1 H_t, \tag{2}$$

where α_0 is a scalar and α_1 is an $N_h \times 1$ vector. We use monthly data, so we adopt the one-month yield as the risk-free rate r_t^f. Following Ang and Piazzesi (2003), the stochastic discount factor (SDF) is given by

$$M_{t+1} = \exp(-r_t^f - \frac{1}{2}x_t^{'}x_t - x_t^{'}\epsilon_{t+1}), \tag{3}$$

where the market price of risk, x_t, follows the affine specification:

$$x_t = \xi_0 + \xi_1 H_t, \tag{4}$$

in which ξ_0 is an $N_h \times 1$ vector, and ξ_1 is an $N_h \times N_h$ matrix. Here the SDF is a quadratic function of x_t. As a result, the total gross return R_{t+1} of any nominal asset satisfies

$$E_t (M_{t+1} R_{t+1}) = 1. \tag{5}$$

For

$$\Theta_0^Q = \Theta_0 - \Sigma_H \xi_0, \tag{6}$$

$$\Theta_1^Q = \Theta_1 - \Sigma_H \xi_1, \tag{7}$$

risk-neutral investor believed that the factors are characterized by a Q-measure VAR given by

$$H_{t+1} = \Theta_0^Q + \Theta_1^Q H_t + \Sigma_H \epsilon_{t+1}^Q, \tag{8}$$

with ϵ_{t+1}^Q a vector of independent standard Normal distribution under the Q-measure. There are three sets of parameters that go into an affine term structure model:

$$(a) \ \Theta_0^Q, \Theta_1^Q, \Sigma_H; \tag{9}$$

$$(b) \ \Theta_0, \Theta_1; \tag{10}$$

$$(c) \ \xi_0, \xi_1. \tag{11}$$

If we know any two of these sets of parameters, we could calculate the third using (6) and (7).

3.2   Treasury Yields

Let $P^{TB}(H_t, \tau)$ denote the price of a τ-period default-free zero coupon bond at time t. That is, $P^{TB}(H_t, \tau)$ denotes the time-t price of an asset with a fixed payoff of one at time $t + \tau$. Because this asset has no intermediate payoffs, its return between t and $t + 1$ is given as

$$R^{TB}(H_{t+1}, \tau) = \frac{P^{TB}(H_{t+1}, \tau - 1)}{P^{TB}(H_t, \tau)}. \tag{12}$$

Then, equation (5) allows bond prices to be computed recursively by

$$P^{TB}(H_t, \tau) = E_t\left[M_{t+1}P^{TB}(H_{t+1}, \tau - 1)\right], \tag{13}$$

with boundary condition $P^{TB}(H_t, 0) = 1$. Therefore, Treasury bond prices are exponential affine functions of the state variables (Ang and Piazzesi (2003)):

$$P^{TB}(H_t, \tau) = \exp\{-A(\tau) - B(\tau)^T H_t\}, \tag{14}$$

where

$$B\left(\tau+1\right) = \alpha_1 + B\left(\tau\right)\Theta_1^Q, \tag{15}$$

$$A\left(\tau+1\right) = \alpha_0 + A\left(\tau\right) + B\left(\tau\right)^T \Theta_0^Q - \frac{1}{2}B\left(\tau\right)^T \Sigma_H^T \Sigma_H B\left(\tau\right), \tag{16}$$

with $B\left(0\right) = 0_{1 \times N_h}$ and $A\left(0\right) = 0$. The continuously compounded spot rates are affine functions of the underlying states,

$$y(H_t, \tau) = -\frac{\log P^{TB}(H_t, \tau)}{\tau} = \frac{A\left(\tau\right)}{\tau} + \frac{B\left(\tau\right)^T}{\tau}H_t. \tag{17}$$

The observed yields on zero coupon Treasury bonds are given by

$$y(t, \tau) = y(H_t, \tau) + e\left(t, \tau\right), \tag{18}$$

where $e\left(t, \tau\right)$ denotes the portion of the spot rate that is not explained by the underlying dynamic factors.

3.3 Corporate Bond Spreads

Duffie and Singleton (1999), and Duffie, Pedersen, and Singleton (2003) show that defaultable bonds can be valued as if they were risk-free by replacing the short rate r_t^f with a default adjusted rate $r_t^f + s_t^i$, where s_t^i for a certain credit-rating class i is an affine function of the underlying factors,

$$s_t^i = \eta_0^i + \eta_1^i H_t + \epsilon_t^i, \tag{19}$$

where ϵ_t^i denotes the portion of the credit spread that is not explained by the underlying factors. In discrete time, the same is true under the recovery of market value assumption.[3] Therefore, we can write the time-t price of a zero-coupon defaultable bond for a certain credit-rating class

---

[3]Previous studies make discretization of continuous-time models, which have served as the building blocks of numerous theoretical models established in the literature. See, for example, Mueller (2008) and Wu and Zhang (2008).

i with time to maturity τ as

$$P_i^{CB}(H_t, \tau) = \exp(-A_i(\tau) - B_i(\tau)^T H_t), \qquad (20)$$

with

$$B_i(\tau + 1) = \alpha_1 + \eta_1^i + B_i(\tau)\Theta_1^Q, \qquad (21)$$

$$A_i(\tau + 1) = \alpha_0 + \eta_0^i + A_i(\tau) + B_i(\tau)^T \Theta_0^Q - \frac{1}{2} B_i(\tau)^T \Sigma_H^T \Sigma_H B_i(\tau), \qquad (22)$$

subject to the boundary conditions $B_i(0) = 0_{1 \times N_h}$ and $A_i(0) = 0$.[4] The continuously compounded spot rates on the defaultable bond is an affine function of the underlying states,

$$y_i(H_t, \tau) = -\frac{\log P_i^{CB}(H_t, \tau)}{\tau} = \frac{A_i(\tau)}{\tau} + \frac{B_i(\tau)^T}{\tau} H_t. \qquad (23)$$

The observed yields on zero coupon defaultable bond are given by

$$y_i(t, \tau) = y_i(H_t, \tau) + e_i(t, \tau), \qquad (24)$$

where $e_i(t, \tau)$ denotes the portion of the spot rate that is not explained by the underlying dynamic factors. Credit spreads can then be calculated as the difference between the yields on defaultable and default-free bonds:

$$
\begin{aligned}
CS_i(t, \tau) &= y_i(t, \tau) - y(t, \tau) \\
&= \left[\frac{A_i(\tau) - A(\tau)}{\tau}\right] + \left[\frac{B_i(\tau) - B(\tau)}{\tau}\right]^T H_t + e_i(t, \tau) - e(t, \tau). \quad (25)
\end{aligned}
$$

This model provides insights into the determinants of the Treasury yields, corporate bond yields, and the credit spreads.

---

[4]For the recursive equations, see Appendix A.

4 Econometric Methodology

4.1 Basic Framework

In this setup, the risk-free rate, r_t^f is assumed to be general affine functions of the underlying state vector:

$$r_t^f = \alpha_0 + \alpha_1 H_t, \tag{26}$$

where α_0 is a scalar and α_1 is an $N_h \times 1$ vector. As a result,

$$r_t^f = \alpha_0 + \alpha_{1m} H_t^m + \alpha_{1l} H_t^l. \tag{27}$$

Since H_t^m is independent of H_t^l, the values of α_0, α_{1m} can be obtained by OLS estimation of

$$r_t^f = \alpha_0 + \alpha_{1m} H_t^m + v_t. \tag{28}$$

To determine the lags of the VAR for the state variables in Eq. (1), we follow Ang and Piazzesi (2003) and regress the short rate on economic factors and 12 lags of economic factors, respectively. The motivations for the two specifications with and without lags in Ang and Piazzesi (2003) stem from the original Taylor rule and the forward-looking version of the Taylor rule. Analogously, our two regressions can be regarded as being based on an augmented Taylor rule with financial market factors and its forward-looking version. As shown in panel A of Table 3, the coefficients on inflation, real activity, and liquidity factors are significant and positive. The coefficient on volatility is insignificant, which suggests that the volatility factor might not be a determinant of the short rate. In contrast, panel B shows that most parameter estimates for the looking-forward version of the augmented Taylor rule are not significant, except for the first lag on real activity and liquidity factors. This outcome suggests that the inclusion of many lags may lead to an overcomplicated and poorly behaved regression model. Therefore, we will use a VAR (1) for both the observable macro variables and the latent variables.

To estimate the model, we first solve for the unobservable factors from the joint dynamics of the economic factors, Treasury yields, and corporate bond yields. We follow Chen and Scott (1993), Ang and Piazzesi (2003), and Hamilton and Wu (2012) and assume that as many yields as unobservable factors are treated as measured without error, and the remaining yields are measured with error. In particular, we assume that the 1- and 12-month Treasury yields and 24-month BBB-rated corporate bond yields are priced without error, while the 3-, 24-, and 36-month Treasury yields and 60- and 120-month corporate bond yields are measured with error. Let Y_t^1 denote the $(N_l \times 1)$ vector consisting of yields without measurement error and Y_t^2 the remaining $(N_e \times 1)$ yields with measurement error. Then, the measurement specification is given as

$$
\begin{bmatrix} Y_t^1 \\ Y_t^2 \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} + \begin{bmatrix} D_{1m} & D_{1l} \\ D_{2m} & D_{2l} \end{bmatrix} \begin{bmatrix} H_t^m \\ H_t^l \end{bmatrix} + \begin{bmatrix} 0 \\ \Sigma_e \end{bmatrix} u_t^e, \tag{29}
$$

where Σ_e is assumed to be diagonal.

We adopt a VAR(1) for both the observable macroeconomic factors, financial market factors, and the latent factors:

$$
H_{t+1}^m = \Theta_{0m} + \Theta_{mm} H_t^m + \Theta_{ml} H_t^l + \Sigma_{mm} \epsilon_{t+1}^m, \tag{30}
$$

$$
H_{t+1}^l = \Theta_{0l} + \Theta_{lm} H_t^m + \Theta_{ll} H_t^l + \Sigma_{lm} \epsilon_{t+1}^m + \Sigma_{ll} \epsilon_{t+1}^l. \tag{31}
$$

Ang and Piazzesi (2003) propose that the macro dynamics are orthogonal to the unobserved latent factors, so that terms such as Θ_{ml} and Θ_{lm} are set to zero. In our setup, the macro dynamics have mean zero by construction, which means Θ_{0m} is set to zero. Ang and Piazzesi (2003) further assumed that:

$$
\Theta_{0l} = 0, \tag{32}
$$

$$
\Sigma_{lm} = 0, \tag{33}
$$

$$
\Sigma_{ll} = I_{N_l}, \tag{34}
$$

and

$$\Sigma_{mm} \text{ is lower triangular,} \tag{35}$$

$$\Theta_{ll} \text{ is lower triangular.} \tag{36}$$

Therefore, the structure model is given as

$$H_t^m = \Theta_{mm} H_{t-1}^m + \Sigma_{mm} \epsilon_t^m \tag{37}$$

$$Y_t^1 = C_1 + D_{1m} H_t^m + D_{1l} H_t^l \tag{38}$$

$$Y_t^2 = C_2 + D_{2m} H_t^m + D_{2l} H_t^l + \Sigma_e u_t^e. \tag{39}$$

4.2 Identification

Following Hamilton and Wu (2012, 2014), we change the structure model to the reduced model.[5] The reduced form is given as

$$H_t^m = \phi_{mm}^* H_{t-1}^m + u_{mt}^* \tag{40}$$

$$Y_t^1 = C_1^* + \phi_{1m}^* H_{t-1}^m + \phi_{11}^* Y_{t-1}^1 + \psi_{1m}^* H_t^m + u_{1t}^* \tag{41}$$

$$Y_t^2 = C_2^* + \phi_{2m}^* H_t^m + \phi_{21}^* Y_t^1 + u_{2t}^*, \tag{42}$$

with

$$\phi_{mm}^* = \Theta_{mm} \tag{43}$$

$$u_{mt}^* = \Sigma_{mm} \epsilon_t^m \tag{44}$$

---

[5]For the details, see Appendix B.

$$C_1^* = C_1 - D_{1l}\Theta_{ll}D_{1l}^{-1}C_1 \tag{45}$$

$$\phi_{1m}^* = -D_{1l}\Theta_{ll}D_{1l}^{-1}D_{1m} \tag{46}$$

$$\phi_{11}^* = D_{1l}\Theta_{ll}D_{1l}^{-1} \tag{47}$$

$$\psi_{1m}^* = D_{1m} \tag{48}$$

$$u_{1t}^* = D_{1l}\epsilon_t^l \tag{49}$$

$$C_2^* = C_2 - D_{2l}D_{1l}^{-1}C_1 \tag{50}$$

$$\phi_{2m}^* = D_{2m} - D_{2l}D_{1l}^{-1}D_{1m} \tag{51}$$

$$\phi_{21}^* = D_{2l}D_{1l}^{-1} \tag{52}$$

$$u_{2t}^* = \Sigma_e u_t^e \tag{53}$$

$$Var\begin{bmatrix} u_{mt}^* \\ u_{1t}^* \\ u_{2t}^* \end{bmatrix} = \begin{bmatrix} \Omega_m^* & & \\ & \Omega_1^* & \\ & & \Omega_2^* \end{bmatrix} = \begin{bmatrix} \Sigma_{mm}\Sigma_{mm}' & & \\ & D_{1l}D_{1l}' & \\ & & \Sigma_e\Sigma_e' \end{bmatrix}. \tag{54}$$

Because u_{mt}^*, u_{1t}^* and u_{2t}^* are independent, full information maximum likelihood estimation is obtained by treating the three blocks separately, and with each block implemented by OLS equation by equation.

$$\widehat{\Omega}_m^* = T^{-1}\sum_{t=1}^{T}\left[\left(H_t^m - \widehat{\phi}_{mm}^* H_{t-1}^m\right)\cdot\left(H_t^m - \widehat{\phi}_{mm}^* H_{t-1}^m\right)^T\right] \tag{55}$$

$$\widehat{\Omega}_1^* = T^{-1}\sum_{t=1}^{T}\left[\begin{array}{c}\left(Y_t^1 - \widehat{C}_1^* - \widehat{\phi}_{1m}^* H_{t-1}^m - \widehat{\phi}_{11}^* Y_{t-1}^1 - \widehat{\psi}_{1m}^* H_t^m\right)\cdot\\ \left(Y_t^1 - \widehat{C}_1^* - \widehat{\phi}_{1m}^* H_{t-1}^m - \widehat{\phi}_{11}^* Y_{t-1}^1 - \widehat{\psi}_{1m}^* H_t^m\right)^T\end{array}\right] \tag{56}$$

$$\widehat{\Omega}_2^* = T^{-1}\sum_{t=1}^{T}\left[\begin{array}{c}\left(Y_t^2 - \widehat{C}_2^* - \widehat{\phi}_{2m}^* H_t^m - \widehat{\phi}_{21}^* Y_t^1\right)\cdot\\ \left(Y_t^2 - \widehat{C}_2^* - \widehat{\phi}_{2m}^* H_t^m - \widehat{\phi}_{21}^* Y_t^1\right)^T\end{array}\right] \tag{57}$$

Table 4 summarizes the mapping between reduced-form and structural parameters. It is worth noting that α_0 cannot be estimated separately from the OLS regression (28), because the risk-free rate serves a dependent variable not only in the regression (28), but also in the regression (38). Another important observation is that the 9 elements of α_0, ξ_0 and η_0 can be

inferred only from the 8 elements of C_1 and C_2. This implies that α_0 and ξ_0 are unidentified. We will discuss how to impose restrictions necessary for identification in Section 4.4.

4.3 Minimum-Chi-Square Estimation

We denote by π the vector consisting of all reduce-form parameters, by $\mathcal{L}(\pi; Y)$ the log likelihood for the entire sample, and by $\widehat{\pi} = \arg\max \mathcal{L}(\pi; Y)$ the vector of full information maximum likelihood (FIML) estimates. We assume that $\widehat{R}$ is a consistent estimate of the information matrix, which satisfies

$$R = -\frac{1}{T} E\left[\frac{\partial^2 \mathcal{L}(\pi; Y)}{\partial\pi\partial\pi'}\right]. \tag{58}$$

Based on estimate of the information matrix, we could then use the Wald test to examine the hypothesis that $\pi = g(\theta)$, where θ is a known vector of parameters. The Wald statistic is calculated as

$$T\left[\widehat{\pi} - g(\theta)\right]' \widehat{R}\left[\widehat{\pi} - g(\theta)\right], \tag{59}$$

and its asymptotic distribution under the null hypothesis follows $\chi^2(q)$, where the degree of freedom q is the dimension of π. As mentioned in Rothenberg (1973), one could also use (59) to choose an estimate $\widehat{\theta}$, which minimizes the chi-square statistic for the estimation. We could obtain asymptotic standard errors by considering an linear approximation

$$g(\theta) \simeq g(\theta_0) + \frac{\partial g(\theta)}{\partial\theta'}|_{\theta=\theta_0}(\theta - \theta_0), \tag{60}$$

with

$$\begin{aligned} \gamma &\equiv g(\theta_0) - \Gamma\theta_0, \\ \Gamma &\equiv \frac{\partial g(\theta)}{\partial\theta'}|_{\theta=\theta_0}, \end{aligned}$$

where $\widehat{\pi} \to \pi$, and we assume there exists a value of θ_0 in the true model such that $g(\theta_0) = \pi_0$. We denote by $\widehat{\theta}^*$ the linearized minimum-chi-square estimator solving the optimization problem

$$\min_\theta \ \mathbf{T}\left[\widehat{\pi} - \gamma - \Gamma\theta\right]' R\left[\widehat{\pi} - \gamma - \Gamma\theta\right]. \tag{61}$$

In other words, $\widehat{\theta}^*$ satisfies

$$\Gamma' R\left(\widehat{\pi} - \gamma - \Gamma\widehat{\theta}^*\right) = 0, \tag{62}$$

or equivalently,

$$\widehat{\theta}^* = \left(\Gamma' R\Gamma\right)^{-1} \Gamma' R\left(\widehat{\pi} - \gamma\right). \tag{63}$$

Then we have

$$\sqrt{T}\left(\widehat{\pi} - \pi_0\right) \to N\left(0, R^{-1}\right), \tag{64}$$

and it implies that

$$\sqrt{T}\left(\widehat{\theta}^* - \theta_0\right) \to N\left(0, \left[\Gamma' R\Gamma\right]^{-1}\right). \tag{65}$$

Therefore, Hamilton and Wu (2012) propose to approximate the variance of $\widehat{\theta}$ with $T^{-1}\left[\widehat{\Gamma}'\widehat{R}\widehat{\Gamma}\right]^{-1}$, in which $\widehat{\Gamma} = \frac{\partial g(\theta)}{\partial \theta'}|_{\theta=\widehat{\theta}}$. They show that this is identical to the usual asymptotic variance for the MLE approach. Put differently, the MCSE and MLE are asymptotically equivalent.

When a model is just identified, the minimum value attainable for (59) is zero. Then the problem in (59) can be simplified as

$$\min_\theta \left[\widehat{\pi} - g(\theta)\right]' \left[\widehat{\pi} - g(\theta)\right]. \tag{66}$$

It is important to note that in the case where the objective function in (66) is equal to zero, estimators resulting from two approaches, i.e., the MCSE and MLE, are asymptotically equivalent. As argued in Hamilton and Wu (2012), the minimum-chi-square algorithm has two major advantages over the traditional brute-force maximization of the likelihood function, although $\widehat{\theta}_{MCSE}$ is identical to $\widehat{\theta}_{MLE}$. First, checking whether (66) equals zero yields an observation on whether $\widehat{\theta}$ corresponds to a global maximum of the original likelihood surface. However, using

the traditional approach requires hundreds of starting values to ensure a global maximum is indeed achieved. A second advantage is that the new optimization problem is computationally much simpler than the original likelihood function.

4.4   Estimation for Macro Finance Model

In this section, we discuss how to estimate our system and how to impose additional restrictions to facilitate the estimation. As discussed in Section 4.2, we have three independent blocks in the reduced-form equations (40)-(42). We assume that $Y_t^m = H_t^m$, and then the structure of block i for $i = 1, 2, m$ can be written in a compact as

$$Y_t^m = \Pi_i^{'} x_{it} + u_{it}^* \tag{67}$$

$$u_{it}^* \sim N\left(0, \Omega_i^*\right). \tag{68}$$

Following Magnus and Neudecker (1988), the information matrix for all reduced-form parameters takes the form as

$$\widehat{R} = \begin{bmatrix} \widehat{R}_m & 0 & 0 \\ 0 & \widehat{R}_1 & 0 \\ 0 & 0 & \widehat{R}_2 \end{bmatrix} \tag{69}$$

where

$$\widehat{R}_i = \begin{bmatrix} \widehat{\Omega}_i^{*-1} \otimes T^{-1}\Sigma_{t=1}^T x_{it}x_{it}' & 0 \\ 0 & \frac{1}{2}D_{qi}'\left(\widehat{\Omega}_i^{*-1} \otimes \widehat{\Omega}_i^{*-1}\right) D_{qi} \end{bmatrix} \tag{70}$$

for D_N the $N^2 \times N(N + 1)/2$ duplication matrix satisfying $D_N \text{vech}(\Omega) = \text{vec}(\Omega)$. The structural parameters Σ_e introducing measurement errors only exist in the block Ω_2^*, and thereby R_2, and no other parameters appear in this block. The 5 structural parameters capturing measurement errors in Σ_e are just-identified by the 5 diagonal elements of $\widehat{\Omega}_2^*$. Using the MCSE approach yields the estimates of diagonal elements of Ω_2^*. It is then easy to obtain the estimates of Σ_e by taking the square roots of the above diagonal elements of $\widehat{\Omega}_2^*$. As proposed by Ang and Piazzesi (2003), $\widehat{\alpha}_{1m}$ can be obtained by implementing the OLS method based on the

assumption that macro variables and latent factors are independent (see equation (28)). Then our system can be estimated by the minimum-chi-square method. The objective function in (59) is given by

$$\widehat{\pi} = \left(\left[\text{vec} \left(\widehat{\Pi}_1 \right) \right]', \left[\text{vech} \left(\widehat{\Omega}_1^* \right) \right]', \left[\text{vec} \left(\widehat{\Pi}_2 \right) \right]' \right)' \tag{71}$$

$$\widehat{R} = \begin{bmatrix} \widehat{\Omega}_1^{*-1} \otimes T^{-1}\Sigma_{t=1}^T x_{1t}x_{1t}' & 0 & 0 \\ 0 & \frac{1}{2}D_3' \left(\widehat{\Omega}_1^{*-1} \otimes \widehat{\Omega}_1^{*-1} \right) D_3 & 0 \\ 0 & 0 & \widehat{\Omega}_2^{*-1} \otimes T^{-1}\Sigma_{t=1}^T x_{2t}x_{2t}' \end{bmatrix} \tag{72}$$

$$x_{1t} = \left(1, H_{t-1}^{m'}, Y_{t-1}^{1'}, H_t^{m'} \right)' \tag{73}$$

$$x_{2t} = \left(1, H_t^{m'}, Y_t^{1'} \right)' \tag{74}$$

$$\widehat{\Pi}_i' = \left(\Sigma_{i=1}^T Y_t^i x_{it}' \right) \left(\Sigma_{i=1}^T x_{it}x_{it}' \right)^{-1} \text{ for } i = 1, 2 \tag{75}$$

$$\widehat{\Omega}_1^* = T^{-1}\Sigma_{i=1}^T \left(Y_t^1 - \widehat{\Pi}_1' x_{1t} \right) \left(Y_t^1 - \widehat{\Pi}_1' x_{1t} \right)' \tag{76}$$

$$\widehat{\Omega}_2^* = T^{-1}\Sigma_{i=1}^T \begin{bmatrix} [\widehat{u}_{2t}(1)]^2 & \ldots & 0 \\ \ldots & & \ldots \\ 0 & \ldots & [\widehat{u}_{2t}(N_e)]^2 \end{bmatrix}, \tag{77}$$

with $\widehat{u}_{2t}(j)$ the jth element of $Y_t^2 - \widehat{\Pi}_2' x_{2t}$.

Following Ang and Piazzesi (2003) and Hamilton and Wu (2012), we impose further restrictions on parameters to achieve identification. The parameters fixed at zero include the (2,1) and (3,1) elements of Θ_{ll} (which was already lower triangular), the (1,2), (2,2), (3,2) and (1,3) elements of ξ_{ll} , all four elements in ξ_{0m}, and the second and third elements of ξ_{0l}. In order to optimize the objective function, we need to calculate the estimates of 37 remaining unknown parameters, 1 in Θ_0^Q, 16 in Θ_{mm}^Q, 5 in Θ_{ll}^Q, 4 in Θ_{ll}, 1 in η_0^{BBB}, 7 in η_1^{BBB}, and 3 in α_{1l}.[6] After getting the parameters of (a) and (b) for the affine structure model, we could

---

[6]Following Ang and Piazzesi (2003) and Hamilton and Wu (2012), we change the parameterization to improve the numerical performance of the MCSE algorithms. Particularly, we impose the restrictions such that Θ_{ll} and Θ_{ll}^Q take the following forms: $\Theta_{ll} = \begin{bmatrix} \theta_1 & 0 & 0 \\ 0 & \theta_2 & 0 \\ 0 & \theta_3 & \theta_4 \end{bmatrix}$, and $\Theta_{ll}^Q = \begin{bmatrix} \theta_5 & 0 & 0 \\ \theta_6 & \theta_2 & \theta_7 \\ \theta_8 & \theta_3 & \theta_9 \end{bmatrix}$.

calculate the parameters of the market price of risk, 1 in ξ_{0l}, 16 in ξ_{1m}, 5 in ξ_{1l}, using 6 and 7. Based on the parameters of (a), (b), and (c), we can numerically solve for the unknown parameters of rated-A corporate bond, 1 in η_0^{BBB}, 7 in η_1^{BBB}, by using equations (21) and (22).

5 Estimation Results

5.1 Unrestricted Estimation

For a preliminary view of how Treasury yields and credit spreads respond to macro factors, we run some unrestricted OLS regressions. Table 5 reports the estimation results from the regressions of Treasury yields with three different maturities (2, 5, and 10 years) on economic factors. Two types of specifications are considered: one only with inflation and real activity as the regressors, and the other one with all four macro factors. The adjusted R^2 of all the estimated models exceeds 35%. These numbers suggest that economic factors should help explain the dynamics of Treasury yields, which is consistent with previous studies. This explanatory power might be lower for long-term bonds, because the adjusted R^2 decreases with the bond maturity. While the coefficients on inflation and real activity factors are significant in all specifications, the coefficient on volatility factor is never significant, and neither is the liquidity factor in the estimation for 10-year Treasury yields. Moreover, adding the latter two regressors only marginally improves the model fit as reflected by the adjusted R^2 in the bottom panel of Table 5 . These facts together imply that the explanatory power of the financial market factors for Treasury yield movements should be limited.

Table 6 and Table 7 present the unrestricted OLS estimation results for the A and BBB credit spreads, respectively. In contrast to the Treasury yield regression results, the inclusion of financial market factors gives rise to a substantial improvement in the goodness of fit, and this effect is more pronounced for the A credit spread than for the BBB credit spread. This highlights the vital role of financial market factors in describing the dynamics of credit spreads,

23

particularly the dynamics of higher-rated credit spreads. The coefficients on the liquidity and volatility factors are highly significant. The coefficient on the liquidity factor is negative, while that on the volatility factor is positive. This is consistent with the previous literature that credit spreads widen as the aggregate market liquidity condition deteriorates and the market as a whole becomes more volatile.[7]

5.2 Estimation with No-Arbitrage Restrictions

Now we turn to the model with no-arbitrage restrictions imposed. The results of the MCSE of Treasury yields and corporate bond yields for 100 different starting values are shown in Table 8. Another advantage of the MCSE is that the value for the objective function itself gives us an immediate test of the various overidentifying restrictions. We have 82 known parameters in the reduced form model in Eq.(71) and 37 unknown parameters in ATSM. As a result, we do have 45 degree of freedom. The 1% critical value for a $\chi^2(45)$ variable is 70. Thus, the observed minimum value (11.25) provides evidence that the restrictions imposed by the model are consistent with the observed data.

The autocorrelations of the three latent factors are large, implying that they are highly persistent. The intercept estimate on the Treasury yields measures the long-term means of the instantaneous Treasury yields, which is estimated at 0.27% and significant. The intercept estimates on the corporate yields measure the fixed component of the instantaneous credit spread, which are 0.36% for A-rated corporate bonds and 0.44% for B-rated corporate bonds. As expected, the compensation for credit risk is higher for higher-rated bonds.

The slope estimates on Treasury yields measure the initial response of the instantaneous Treasury yields to unit shocks on the seven factors. The estimates for the macro factors are positive and are larger than those for the financial market factors, suggesting that the latter variables only play a marginal role in the dynamics of the short Treasury yields. The absolute magnitude of the slope estimates on the short spread gets much higher for financial market

---

[7]See Wu and Zhang (2008) for the effect of financial market volatility on credit spreads and Lin, Wang, and Wu (2011) for the effect of liquidity.

factors. In particular, the liquidity factor even becomes the most important driver of the short spread, and this variation is more pronounced for the A-rated bonds. Thus, in contrast to the finding of Wu and Zhang (2008) that financial market volatility factor is a major determinant of the credit spread term structure, our results show that the liquidity factor is more important than the volatility factor for capturing the dynamics of credit spreads.

5.3 Factor Loadings and Impulse Responses

Figure 5 illustrates how Treasury yields and corporate yields respond to a one standard deviation change in macro factors. As shown in panel (a), the loadings on inflation are positive for Treasury yields but negative for corporate yields, implying that positive inflation shocks lead to higher Treasury yields but lower corporate yields. For a certain maturity, the absolute magnitudes of inflation loadings are larger for corporate yields than for Treasury yields. Thus, the former is contemporaneously more sensitive to inflation change than the latter. Furthermore, it declines with maturity for both Treasury and corporate yields and exhibits a convergence to the long-run mean. Hence, long yields are less subject to inflation change than short yields. Similar patterns are observed for the real activity loadings in panel (b). One primary distinction is that the real activity loadings of the BBB corporate yields become larger than those of the A corporate yields and intensify with maturity.

Figure 6 shows the loadings of Treasury yields and corporate yields on financial market factors. Treasury yields have rather small loadings on both the liquidity and volatility factors. This is consistent with the unrestricted estimation results that Treasury yields are more influenced by macro factors than by financial market factors. The loadings on liquidity for both the BBB and A corporate yields are negative and decline in absolute value with maturity. Moreover, the liquidity loadings are larger in absolute value for the A corporate yields than for the BBB corporate yields, which suggests that the initial reaction to funding liquidity shocks is stronger for higher-rated bonds. As shown in panel (b), the loadings on the volatility factor differ between the two corporate yields; whereas the volatility loadings of the BBB corporate

25

yields are largely flat and remain positive across bond maturities, those of the A corporate yields are negative when the maturity is short and display an upward trend. This implies that for A-rated bonds, short and long yields react to the volatility change in different directions.

With factor loadings of Treasury yields and corporate yields, one can easily derive the factor loadings of credit spreads using equation (25). The results are shown in Figure 7 and Figure 8, which include economic factor loadings and latent factor loadings, respectively. The inflation, real activity and liquidity loadings of both the A and BBB credit spreads are high when the maturity is short and converge fast to their long term means, while the volatility loadings increase with maturity. Thus, the contemporaneous reaction to economic factors is generally larger for short spreads than for long spreads, except for the volatility factor. In contrast to Treasury yields, credit spreads load heavily on both the liquidity and volatility factors. This is consistent with the unrestricted regression results and justifies the importance of those factors for explaining credit spreads. The liquidity loadings are negative and higher in absolute value for the A credit spreads. This implies that funding liquidity shrinkage widens the credit spreads and this effect is more pronounced for those at a high-rating class. The observation that the volatility factor generally exerts a positive effect on the credit spread term structure is consistent with Wu and Zhang (2008). However, this effect is reversed for short A credit spreads. As shown in Figure 8, the loadings on the three latent factors behave in a similar fashion for both the A and BBB credit spreads.

Figure 9 illustrates the impulse response functions of 24-, 60- and 120-month Treasury yields. The impulse responses for all factors are large in absolute value at short horizon and level off slowly towards zero. The impulse response for inflation and real activity remains positive even after 12 months, whereas those for liquidity and volatility quickly fade away within one year; this effect is more pronounced for long-term Treasury yields. This implies that macroeconomic shocks have a more persistent impact on Treasury yields than do financial market shocks. The transient effects of financial market shocks are quite large: shocks to the volatility factor have even larger impacts on long-term Treasury yields than those to macro factors at the very short horizon. Figure 10 and Figure 11 plot the impulse response

functions of 24-, 60- and 120-month A and BBB credit spreads, respectively. Liquidity has larger instantaneous effects on the A credit spreads than on the BBB credit spreads. For all corporate bonds under consideration, the initial response of their credit spreads to financial market factors is much larger than that to macro factors. However, in general, this relationship is reversed after one year, implying that macro factors have more persistent effects on the credit spread term structure compared to financial market factors.

5.4 Forecasts

A true out-of-sample analysis would be to estimate the model based on data up to and including today, construct a forecast of tomorrow's value at $T + 1$, wait until tomorrow, record the forecast error at $T + 1$, re-estimate the model, make a new forecast of true value at $T + 2$, and so forth. At the end of this exercise, one would have a sample of forecast errors which would be truly out-of-sample and would give a very realistic picture of the model's performance. Since this procedure is very time-consuming, we instead resort to "pseudo," or "simulated," out-of-sample analysis, which means to mimic the procedure described above, using some historical date $T_0 < T$, rather than today's date T, as a starting point. The resulting forecasting errors are then used to assess the model's out-of-sample forecasting ability.

We perform a comparison of out-of-sample forecasts for credit spreads for three models. First, we investigate out-of-sample forecasts for two VAR(1) models without cross-equation restrictions induced by no-arbitrage condition. The first VAR involves two macro factors (inflation and real activity), and the second incorporates both macro factors and financial market factors (liquidity and volatility). Our last model is a VAR(1) model with four economic factors and cross-equation restrictions. We forecast over the last 36 months of our sample, which is chosen to avoid forecasting abnormal yields in recent financial crisis. We record the root mean square error (RMSE) and the mean absolute deviation (MAD) of the forecast versus the actual value as criteria for comparing the forecasting abilities of different models.

Table 9 shows the comparisons of the out-of-sample forecasts. Lower RMSE and MAD

denote better forecasts. The best forecasts are highlighted in bold. Two important points are noteworthy. First, incorporating financial market variables improves forecasts. In terms of both the RMSE and MAD criteria, the models with four factors uniformly outperformed the model with only macro factors (except for the 120-month A credit spreads). The improvement in forecasting performance is much more noticeable for short-term credit spreads, which is consistent with the previous result that the explanatory power of financial market factors for credit spreads emerges primarily at the short end of the yield curve. Second, imposing the cross-equation restrictions induced by no-arbitrage improves forecasts. This is strongly supported by the results of the RMSE: the no-arbitrage model outperforms the other two unrestricted models (except for 24-month A credit spreads). However, less evidence is found in results of the MAD, as the no-arbitrage model provides the best forecast only for the 24-month BBB credit spreads. This mismatch is likely due to the fact that the RMSE penalizes large errors more than the MAD does. Given that the MADs for the no-arbitrage model are close to those for the four-factor model and large errors are undesirable in our context, we can still conclude that adding term structure restrictions helps in forecasting.

6 Conclusion

This paper presents and estimates a no-arbitrage Gaussian affine term structure model of interest rate and credit spreads with observable economic variables and traditional latent variables. In addition to macro variables, financial market variables are included in the set of observable variables. The estimation is performed using a minimum-chi-square method, which is much more efficient and reliable than the widespread MLE. By estimating the model, we quantify the effects of economic variables on the whole term structure of interest rates and credit spreads and explore their role in forecasting.

We find that macro factors are important determinants of both Treasury yields and credit spreads. Positive inflation and real activity shocks increase Treasury yields but decrease credit spreads. In contrast, financial market factors only have marginal effects on the Treasury yield

curve but exert substantial impacts on the credit spread term structure. The volatility factor has positive effects on the BBB credit spreads. Its effects on the A credit spreads are negative at the short end of the credit spreads curve but turn positive at the middle. Funding liquidity shrinkage remarkably widens credit spreads, and this effect is strongest at the short end of the credit spread term structure. We show that adding financial market factors substantially improves forecasts for credit spreads. Imposing no-arbitrage restrictions further helps in out-of-sample forecasts.

A   Discretization

A.1   State Variables

Following Wu and Zhang (2008), we consider a flexible specification, under which the factor dynamics under the risk-neutral Q-measure,

$$dH_t = \kappa^Q \left(\theta^Q - H_t \right) dt + \Sigma_H dW_t^Q, \tag{78}$$

where H_t is an $N_h \times 1$ vector of state variables at time t. Under this specification, the variable that determines the price of risk, x_t, is assumed to be general affine functions of the underlying state vector:

$$x_t = \xi_0 + \xi_1 H_t, \tag{79}$$

where ξ_0 is a scalar.

As a result,

$$
\begin{aligned}
H_{t+1} - H_t &= \kappa^Q \left(\theta^Q - H_t \right) + \Sigma_H \epsilon_{t+1}^Q, \\
H_{t+1} &= \kappa^Q \theta^Q + \left(I - \kappa^Q \right) H_t + \Sigma_H \epsilon_{t+1}^Q.
\end{aligned}
$$

Let $\kappa^Q \theta^Q \equiv \Theta_0^Q$, $\left(I - \kappa^Q \right) \equiv \Theta_1^Q$, then

$$H_{t+1} = \Theta_0^Q + \Theta_1^Q H_t + \Sigma_H \epsilon_{t+1}^Q. \tag{80}$$

A.2   Term Structure of Treasury Yields

In our model, the risk-free rate, r_t^f, is assumed to be a general affine functions of the underlying state vector:

$$r_t^f = \alpha_0 + \alpha_1 H_t + \epsilon_t^r, \tag{81}$$

where α_0 is a scalar and ϵ_t^r denotes movements in the instantaneous interest rate that are not explained by the underlying dynamic factors. We can write the time-t price of a default-free zero coupon bond with time to maturity τ as

$$
\begin{aligned}
P^{TB}(t,\tau) &= E_t^Q\left[\exp\left(-\int_t^{t+\tau} r_s^f ds\right)\right] \\
&= E_t^Q\left[\exp\left(-\int_t^{t+\tau}(\alpha_0 + \alpha_1 H_s)\,ds\right)\right] \cdot E_t^Q\left[\exp\left(-\int_t^{t+\tau}\epsilon_s^r ds\right)\right] \\
&= P^{TB}(H_t,\tau)E(t,\tau),
\end{aligned}
\tag{82}
$$

where $E_t^Q[\cdot]$ denotes the expectation under Q-measure conditional on time-t filtration F_t.

Under this specification, $P^{TB}(H_t,\tau)$ is the exponential affine function of the state variables (Ang and Piazzesi (2003)):

$$
P^{TB}(H_t,\tau) = \exp\{-A(\tau) - B(\tau)^T H_t\},
\tag{83}
$$

where the coefficients $A(\tau)$ and $B(\tau)$ are solutions to the following ordinary differential equations:

$$
\begin{aligned}
B'(\tau) &= \alpha_1 - \left(\kappa^Q\right)^T B(\tau), \\
A'(\tau) &= \alpha_0 + B(\tau)^T \kappa^Q \theta^Q - \frac{1}{2}B(\tau)^T \Sigma_H^T \Sigma_H B(\tau),
\end{aligned}
\tag{84}
\tag{85}
$$

with $B(0) = 0_{1\times N_h}$ and $A(0) = 0$.

As a result,

$$
\begin{aligned}
B(\tau+1) - B(\tau) &= \alpha_1 - \left(\kappa^Q\right)^T B(\tau), \\
B(\tau+1) &= \alpha_1 + \left[I - \left(\kappa^Q\right)^T\right]B(\tau), \\
B(\tau+1) &= \alpha_1 + B(\tau)\Theta_1^Q.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
B\left(\tau+1\right) & = \alpha_1 \left(\left(\Theta_1^Q\right)^\tau + \left(\Theta_1^Q\right)^\tau + ... + \Theta_1^Q + 1\right) \\
& = \alpha_1 \left(\frac{\left(\Theta_1^Q\right)^{\tau+1} - 1}{\Theta_1^Q - 1}\right),
\end{aligned}
\tag{86}
$$

and

$$
\frac{B\left(\tau+1\right)}{\tau+1} = \frac{\alpha_1}{\tau+1} \left(\frac{\left(\Theta_1^Q\right)^{\tau+1} - 1}{\Theta_1^Q - 1}\right).
\tag{87}
$$

Also, we have

$$
\begin{aligned}
A\left(\tau+1\right) - A\left(\tau\right) & = \alpha_0 + B\left(\tau\right)^T \kappa^Q \theta^Q - \frac{1}{2} B\left(\tau\right)^T \Sigma_H^T \Sigma_H B\left(\tau\right), \\
A\left(\tau+1\right) - A\left(\tau\right) & = \alpha_0 + B\left(\tau\right)^T \Theta_0^Q - \frac{1}{2} B\left(\tau\right)^T \Sigma_H^T \Sigma_H B\left(\tau\right).
\end{aligned}
\tag{88}
$$

Indeed,

$$
A\left(1\right) = \alpha_0,
\tag{89}
$$

$$
A(2) = \alpha_0 + A(1) + B\left(1\right)^T \Theta_0^Q - \frac{1}{2} B\left(1\right)^T \Sigma_H^T \Sigma_H B\left(1\right),
\tag{90}
$$

$$
\begin{aligned}
A(3) & = \alpha_0 + A(2) + B\left(2\right)^T \Theta_0^Q - \frac{1}{2} B\left(2\right)^T \Sigma_H^T \Sigma_H B\left(2\right) \\
& = 3\alpha_0 + \left(B\left(1\right)^T + B\left(2\right)^T\right) \Theta_0^Q \\
& \quad - \frac{1}{2} \left[B\left(1\right)^T \Sigma_H^T \Sigma_H B\left(1\right) + B\left(2\right)^T \Sigma_H^T \Sigma_H B\left(2\right)\right],
\end{aligned}
\tag{91}
$$

therefore,

$$
\begin{aligned}
A\left(\tau+1\right) & = (\tau+1)\alpha_0 + \left(B\left(1\right)^T + ... + B\left(\tau\right)^T\right) \Theta_0^Q \\
& \quad - \frac{1}{2} \left[B\left(1\right)^T \Sigma_H^T \Sigma_H B\left(1\right) + ... + B\left(\tau\right)^T \Sigma_H^T \Sigma_H B\left(\tau\right)\right],
\end{aligned}
\tag{92}
$$

32

and

$$\frac{A(\tau+1)}{\tau+1} = \alpha_0 + \frac{1}{\tau+1}\left(B(1)^T + ... + B(\tau)^T\right)\Theta_0^Q$$
$$-\frac{1}{2}\frac{1}{\tau+1}\left[B(1)^T\Sigma_H^T\Sigma_H B(1) + ... + B(\tau)^T\Sigma_H^T\Sigma_H B(\tau)\right]. \quad (93)$$

The continuously compounded spot rates are affine functions of the underlying states:

$$y(H_t,\tau) = -\frac{\log P^{TB}(H_t,\tau)}{\tau} = \frac{A(\tau)}{\tau} + \frac{B(\tau)^T}{\tau}H_t. \quad (94)$$

The observed yields on zero coupon Treasury bonds are given by

$$y(t,\tau) = y(H_t,\tau) + e(t,\tau), \quad (95)$$

where $e(t,\tau) = -\ln(E(t,\tau))/\tau$ denotes the portion of the spot rate that is not explained by the underlying dynamic factors.

A.3   Term Structure of Corporate Yields and Credit Spreads

Duffie and Singleton (1999), and Duffie, Pedersen, and Singleton (2003) show that defaultable bonds can be valued as if they were risk-free by replacing the short rate r_t^f with a default adjusted rate $r_t^f + s_t$, where s_t^i for a certain credit-rating class i is an affine function of the underlying factors,

$$s_t^i = \eta_0^i + \eta_1^i H_t + \epsilon_t^i, \quad (96)$$

where ϵ_t^i denotes the portion of the credit spread that is not explained by the underlying factors. Therefore, we can write the time-t price of a zero-coupon defaultable bond for a certain credit-

rating class i with time to maturity τ as

$$
\begin{aligned}
P_i^{CB}(t,\tau) &= E_t^Q\left[\exp\left(-\int_t^{t+\tau}\left(r_u^f + s_u^i\right)du\right)\right] \\
&= E_t^Q\left[\exp\left(-\int_t^{t+\tau}\left(\alpha_0 + \alpha_1 H_u + \eta_0^i + \eta_1^i H_u\right)du\right)\right] \cdot E_t^Q\left[\exp\left(-\int_t^{t+\tau}\left(\epsilon_u^r + \epsilon_u^i\right)du\right)\right] \\
&= P_i^{CB}(H_t,\tau)E_i(t,\tau),
\end{aligned}
\tag{97}
$$

with

$$
P_i^{CB}(H_t,\tau) = \exp(-A_i(\tau) - B_i(\tau)^T H_t),
$$

where the coefficients $A_i(\tau)$, and $B_i(\tau)$ are solutions to the ordinary differential equations:

$$
B_i'(\tau) = \alpha_1 + \eta_1^i - \left(\kappa^Q\right)^T B_i(\tau), \tag{98}
$$

$$
A_i'(\tau) = \alpha_0 + \eta_0^i + B_i(\tau)^T \kappa^Q \theta^Q - \frac{1}{2}B_i(\tau)^T \Sigma_H^T \Sigma_H B_i(\tau), \tag{99}
$$

subject to the boundary conditions $B_i(0) = 0_{1\times N_h}$ and $A_i(0) = 0$. As a result,

$$
\begin{aligned}
B_i(\tau+1) - B_i(\tau) &= \alpha_1 + \eta_1^i - \left(\kappa^Q\right)^T B_i(\tau), \\
B_i(\tau+1) &= \alpha_1 + \eta_1^i + \left[I - \left(\kappa^Q\right)^T\right] B_i(\tau), \\
B_i(\tau+1) &= \alpha_1 + \eta_1^i + B_i(\tau)\Theta_1^Q.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
B_i(\tau+1) &= \left(\alpha_1 + \eta_1^i\right)\left(\left(\Theta_1^Q\right)^\tau + \left(\Theta_1^Q\right)^\tau + ... + \Theta_1^Q + 1\right) \\
&= \left(\alpha_1 + \eta_1^i\right)\left(\frac{\left(\Theta_1^Q\right)^{\tau+1} - 1}{\Theta_1^Q - 1}\right),
\end{aligned}
\tag{100}
$$

and

$$
\frac{B_i(\tau+1)}{\tau+1} = \frac{\left(\alpha_1 + \eta_1^i\right)}{\tau+1}\left(\frac{\left(\Theta_1^Q\right)^{\tau+1} - 1}{\Theta_1^Q - 1}\right). \tag{101}
$$

Also, we have

$$A_i(\tau+1) - A_i(\tau) = \alpha_0 + \eta_0^i + B_i(\tau)^T \kappa^Q \theta^Q - \frac{1}{2} B_i(\tau)^T \Sigma_H^T \Sigma_H B_i(\tau),$$

$$A_i(\tau+1) - A_i(\tau) = \alpha_0 + \eta_0^i + B_i(\tau)^T \Theta_0^Q - \frac{1}{2} B_i(\tau)^T \Sigma_H^T \Sigma_H B_i(\tau). \tag{102}$$

Indeed,

$$A_i(1) = \alpha_0 + \eta_0^i, \tag{103}$$

$$A_i(2) = \alpha_0 + \eta_0^i + A_i(1) + B_i(1)^T \Theta_0^Q - \frac{1}{2} B_i(1)^T \Sigma_H^T \Sigma_H B_i(1), \tag{104}$$

$$A_i(3) = \alpha_0 + \eta_0^i + A_i(2) + B_i(2)^T \Theta_0^Q - \frac{1}{2} B_i(2)^T \Sigma_H^T \Sigma_H B_i(2)$$

$$= 3\left(\alpha_0 + \eta_0^i\right) + \left(B_i(1)^T + B_i(2)^T\right) \Theta_0^Q$$

$$- \frac{1}{2}\left[B_i(1)^T \Sigma_H^T \Sigma_H B_i(1) + B_i(2)^T \Sigma_H^T \Sigma_H B_i(2)\right]. \tag{105}$$

Therefore,

$$A_i(\tau+1) = (\tau+1)\left(\alpha_0 + \eta_0^i\right) + \left(B_i(1)^T + ... + B_i(\tau)^T\right)\Theta_0^Q$$

$$- \frac{1}{2}\left[B_i(1)^T \Sigma_H^T \Sigma_H B_i(1) + ... + B_i(\tau)^T \Sigma_H^T \Sigma_H B_i(\tau)\right], \tag{106}$$

and

$$\frac{A_i(\tau+1)}{\tau+1} = \left(\alpha_0 + \eta_0^i\right) + \frac{1}{\tau+1}\left(B_i(1)^T + ... + B_i(\tau)^T\right)\Theta_0^Q$$

$$- \frac{1}{2}\frac{1}{\tau+1}\left[B_i(1)^T \Sigma_H^T \Sigma_H B_i(1) + ... + B_i(\tau)^T \Sigma_H^T \Sigma_H B_i(\tau)\right]. \tag{107}$$

The continuously compounded spot rates on the defaultable bond is an affine function of the underlying states,

$$y_i(H_t,\tau) = -\frac{\log P_i^{CB}(H_t,\tau)}{\tau} = \frac{A_i(\tau)}{\tau} + \frac{B_i(\tau)^T}{\tau} H_t. \tag{108}$$

The observed yields on zero coupon defaultable bond are given by

$$y_i(t, \tau) = y_i(H_t, \tau) + e_i(t, \tau), \tag{109}$$

where $e_i(t, \tau) = -\ln(E_i(t, \tau))/\tau$ denotes the portion of the spot rate that is not explained by the underlying dynamic factors. Credit spreads can then be calculated as the difference between the yields on defaultable and default-free bonds:

$$
\begin{aligned}
CS_i(t, \tau) &= y_i(t, \tau) - y(t, \tau) \\
&= \left[\frac{A_i(\tau) - A(\tau)}{\tau} \right] + \left[\frac{B_i(\tau) - B(\tau)}{\tau} \right]^T H_t + e_i(t, \tau) - e(t, \tau). \tag{110}
\end{aligned}
$$

This model provides insights into the determinants of the Treasury yields, corporate bond yields, and the credit spreads.

B Estimation

In our model setup, the structure model is given as

$$
\begin{aligned}
H_t^m &= \Theta_{mm} H_{t-1}^m + \Sigma_{mm} \epsilon_t^m \tag{111} \\
Y_t^1 &= C_1 + D_{1m} F_t^m + D_{1l} H_t^l \tag{112} \\
Y_t^2 &= C_2 + D_{2m} F_t^m + D_{2l} H_t^l + \Sigma_e u_t^e \tag{113}
\end{aligned}
$$

Then we have

$$H_t^m = \phi_{mm}^* H_{t-1}^m + u_{mt}^* \tag{114}$$

with

$$
\begin{aligned}
\phi_{mm}^* &= \Theta_{mm}, \tag{115} \\
u_{mt}^* &= \Sigma_{mm} \epsilon_t^m. \tag{116}
\end{aligned}
$$

We substitute $H_t^l = D_{1l}^{-1}\left(Y_t^1 - C_1 - D_{1m}H_t^m\right)$ into the equation (113) for Y_t^2,

$$
\begin{aligned}
Y_t^2 &= C_2 + D_{2m}H_t^m + D_{2l}H_t^l + \Sigma_e u_t^e \\
&= C_2 + D_{2m}H_t^m + D_{2l}D_{1l}^{-1}\left(Y_t^1 - C_1 - D_{1m}H_t^m\right) + \Sigma_e u_t^e \\
&= C_2 - D_{2l}D_{1l}^{-1}C_1 + \left(D_{2m} - D_{2l}D_{1l}^{-1}D_{1m}\right)H_t^m + D_{2l}D_{1l}^{-1}Y_t^1 + \Sigma_e u_t^e. \quad (117)
\end{aligned}
$$

That is,

$$
\begin{aligned}
Y_t^2 &= C_2^* + \phi_{2m}^* H_t^m + \phi_{21}^* Y_t^1 + u_{2t}^* &(118) \\
C_2^* &= C_2 - D_{2l}D_{1l}^{-1}C_1 &(119) \\
\phi_{2m}^* &= D_{2m} - D_{2l}D_{1l}^{-1}D_{1m} &(120) \\
\phi_{21}^* &= D_{2l}D_{1l}^{-1} &(121) \\
u_{2t}^* &= \Sigma_e u_t^e / &(122)
\end{aligned}
$$

Also, we have

$$
\begin{aligned}
Y_t^1 &= C_1 + D_{1m}H_t^m + D_{1l}H_t^l \\
&= C_1 + D_{1m}H_t^m + D_{1l}(\Theta_{ll}H_{t-1}^l + \Sigma_{ll}\epsilon_t^l) \\
&= C_1 + D_{1m}H_t^m + D_{1l}(\Theta_{ll}D_{1l}^{-1}\left(Y_{t-1}^1 - C_1 - D_{1m}H_{t-1}^m\right) + \Sigma_{ll}\epsilon_t^l) \\
&= C_1 - D_{1l}\Theta_{ll}D_{1l}^{-1}C_1 + D_{1m}H_t^m + D_{1l}\Theta_{ll}D_{1l}^{-1}Y_{t-1}^1 - D_{1l}\Theta_{ll}D_{1l}^{-1}D_{1m}H_{t-1}^m + D_{1l}\epsilon_t^l.
\end{aligned}
$$

That is,

$$Y_t^1 = C_1^* + \phi_{1m}^* H_{t-1}^m + \phi_{11}^* Y_{t-1}^1 + \psi_{1m}^* H_t^m + u_{1t}^* \tag{123}$$

$$C_1^* = C_1 - D_{1l}\Theta_{ll}D_{1l}^{-1}C_1 \tag{124}$$

$$\phi_{1m}^* = -D_{1l}\Theta_{ll}D_{1l}^{-1}D_{1m} \tag{125}$$

$$\phi_{11}^* = D_{1l}\Theta_{ll}D_{1l}^{-1} \tag{126}$$

$$\psi_{1m}^* = D_{1m} \tag{127}$$

$$u_{1t}^* = D_{1l}\epsilon_t^l. \tag{128}$$

As a result, the reduced form is given as

$$H_t^m = \phi_{mm}^* H_{t-1}^m + u_{mt}^* \tag{129}$$

$$Y_t^1 = C_1^* + \phi_{1m}^* H_{t-1}^m + \phi_{11}^* Y_{t-1}^1 + \psi_{1m}^* H_t^m + u_{1t}^* \tag{130}$$

$$Y_t^2 = C_2^* + \phi_{2m}^* H_t^m + \phi_{21}^* Y_t^1 + u_{2t}^* \tag{131}$$

$$Var \begin{bmatrix} u_{mt}^* \\ u_{1t}^* \\ u_{2t}^* \end{bmatrix} = \begin{bmatrix} \Omega_m^* & & \\ & \Omega_1^* & \\ & & \Omega_2^* \end{bmatrix} = \begin{bmatrix} \Sigma_{mm}\Sigma_{mm}' & & \\ & D_{1l}D_{1l}' & \\ & & \Sigma_e\Sigma_e' \end{bmatrix}. \tag{132}$$

Because u_{mt}^*, u_{1t}^* and u_{2t}^* are independent, full-information-maximum likelihood estimation is obtained by treating the three blocks separately, and wit each block implemented by OLS equation by equation.

$$\widehat{\Omega}_m^* = T^{-1}\sum_{t=1}^{T}\left[\left(H_t^m - \widehat{\phi}_{mm}^* H_{t-1}^m\right) \cdot \left(H_t^m - \widehat{\phi}_{mm}^* H_{t-1}^m\right)^T\right] \tag{133}$$

$$\widehat{\Omega}_1^* = T^{-1}\sum_{t=1}^{T}\left[\begin{array}{l}\left(Y_t^1 - \widehat{C}_1^* - \widehat{\phi}_{1m}^* H_{t-1}^m - \widehat{\phi}_{11}^* Y_{t-1}^1 - \widehat{\psi}_{1m}^* H_t^m\right) \cdot \\ \left(Y_t^1 - \widehat{C}_1^* - \widehat{\phi}_{1m}^* H_{t-1}^m - \widehat{\phi}_{11}^* Y_{t-1}^1 - \widehat{\psi}_{1m}^* H_t^m\right)^T\end{array}\right] \tag{134}$$

$$\widehat{\Omega}_2^* = T^{-1}\sum_{t=1}^{T}\left[\begin{array}{l}\left(Y_t^2 - \widehat{C}_2^* - \widehat{\phi}_{2m}^* H_t^m - \widehat{\phi}_{21}^* Y_t^1\right) \cdot \\ \left(Y_t^2 - \widehat{C}_2^* - \widehat{\phi}_{2m}^* H_t^m - \widehat{\phi}_{21}^* Y_t^1\right)^T\end{array}\right]. \tag{135}$$

C Log Likelihood for the Macro Model

The P dynamics can be written as a special case of a first-order VAR by using the companion form $H_t = \left[H_t^{m'}, H_t^{l''} \right]'$, $\Theta_0 = \left[0_{4 \times 1}', \Theta_{0l}' \right]'$, and

$$\Theta_1 = \begin{bmatrix} \Theta_{mm} & & 0 \\ (4 \times 4) & & \\ & & \Theta_{ll} \\ 0 & & (3 \times 3) \end{bmatrix} \tag{136}$$

$$\Sigma_H = \begin{bmatrix} \Sigma_{mm} & & 0 \\ (4 \times 4) & & \\ & & \Sigma_{ll} \\ 0 & & (3 \times 3) \end{bmatrix}. \tag{137}$$

Ang and Piazzesi (2003) and Hamilton and Wu (2012) imposed the restriction in the market price of risk, x_t, that the parameters in the affine specification are characterized by $\xi_0 = \left[\xi_{0m}', \xi_{0l}' \right]'$ and

$$\underset{(7 \times 7)}{\xi_1} = \begin{bmatrix} \xi_{1m} & & 0 \\ (4 \times 4) & & \\ & & \xi_{1l} \\ 0 & & (3 \times 3) \end{bmatrix}. \tag{138}$$

From

$$\Theta_0^Q = \Theta_0 - \Sigma_H \xi_0, \tag{139}$$

$$\Theta_1^Q = \Theta_1 - \Sigma_H \xi_1, \tag{140}$$

we can have $\Theta_0^Q = \left[\Theta_{0m}^{Q'}, \Theta_{0l}^{Q'}\right]'$ and

$$
\Theta_1^Q = \begin{bmatrix} \Theta_{mm}^Q & & 0 \\ (4 \times 4) & & \\ & & \Theta_{ll}^Q \\ 0 & & (3 \times 3) \end{bmatrix}. \tag{141}
$$

In our setup, we use $N_l = 3$ and $N_e = 5$, assuming that the 1-, and 12-month Treasury yields and 24-month corporate bond yields are priced without error, while the 3-, 24-, and 36-month Treasury yields and 60- and 120-month corporate bond yields are priced with error, so that the D matrices can be written in partitioned form as

$$
\begin{bmatrix} D_{1m}^0 & D_{1l} \\ (3 \times 4) & (3 \times 3) \\ D_{2m}^0 & D_{2l} \\ (5 \times 4) & (5 \times 3) \end{bmatrix} = \begin{bmatrix} B(1)^T/1 \\ B(12)^T/12 \\ B_i(24)^T/24 \\ B(3)^T/3 \\ B(24)^T/24 \\ B(36)^T/36 \\ B_i(60)^T/60 \\ B_i(120)^T/120 \end{bmatrix}. \tag{142}
$$

The conditional density for this case is given by

$$
f\left(H_t^m, Y_t | H_{t-1}^m, Y_{t-1}\right) = \frac{1}{|\det(J)|} f\left(H_t^m, H_t^l, u_t^e | H_{t-1}^m, H_{t-1}^l, u_{t-1}^e\right). \tag{143}
$$

where

$$
f\left(H_t^m, H_t^l, u_t^e | H_{t-1}^m, H_{t-1}^l, u_{t-1}^e\right) = f\left(H_t^m | H_{t-1}^m\right) f\left(H_t^l | H_{t-1}^l\right) f\left(u_t^e\right), \tag{144}
$$

$$
f\left(H_t^m | H_{t-1}^m\right) = \phi\left(H_t^m; \Theta_{mm} H_{t-1}^m, \Sigma_{mm} \Sigma_{mm}'\right), \tag{145}
$$

$$
f\left(H_t^l | H_{t-1}^l\right) = \phi\left(H_t^l; \Theta_{ll} H_{t-1}^l, \Sigma_{ll} \Sigma_{ll}'\right), \tag{146}
$$

40

$$f\left(u_t^e\right) = \phi\left(u_t^e; 0, I_{N_e}\right), \tag{147}$$

$$\phi\left(y; \mu, \Omega\right) = \frac{1}{(2\pi)^{M/2}|\Omega|^{1/2}} \exp\left[-\frac{(y-\mu)^{'}\Omega^{-1}(y-\mu)}{2}\right] \tag{148}$$

$$H_t^l = D_{1l}^{-1}(Y_t^1 - C_1 - D_{1m}H_t^m) \tag{149}$$

$$u_t^e = \Sigma_e^{-1}(Y_t^2 - C_2 - D_{2m}H_t^m - D_{2l}H_t^l) \tag{150}$$

$$J = \begin{bmatrix} D_{1l} & 0 \\ D_{2l} & \Sigma_e \end{bmatrix}. \tag{151}$$

The traditional approach (Ang and Piazzesi (2003)) is to arrive at estimates of these parameters by numerical maximization of

$$L(\theta; Y) = \sum_{t=1}^{T} \log f\left(H_t^m, Y_t | H_{t-1}^m, Y_{t-1}\right). \tag{152}$$

References

Acharya, Viral V, Yakov Amihud, and Sreedhar T Bharath, 2010, Liquidity risk of corporate bond returns: A conditional approach, Discussion paper, National Bureau of Economic Research.

Altman, Edward I, Brooks Brady, Andrea Resti, and Andrea Sironi, 2005, The link between default and recovery rates: Theory, empirical evidence, and implications, *Journal of Business* 78, 2203–2228.

Ang, Andrew, and Monika Piazzesi, 2003, A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables, *Journal of Monetary Economics* 50, 745–787.

Bekaert, Geert, Seonghoon Cho, and Antonio Moreno, 2010, New keynesian macroeconomics and the term structure, *Journal of Money, Credit, and Banking* 42, 33–62.

Bikbov, Ruslan, and Mikhail Chernov, 2010, No-arbitrage macroeconomic determinants of the yield curve, *Journal of Econometrics* 159, 166–182.

Boyson, Nicole M, Christof W Stahel, and Rene M Stulz, 2010, Hedge fund contagion and liquidity shocks, *The Journal of Finance* 65, 1789–1816.

Brunnermeier, Markus K., and Lasse Heje Pedersen, 2009, Market liquidity and funding liquidity, *Review of Financial Studies* 22, 2201–2238.

Carey, Mark, 1998, Credit risk in private debt portfolios, *The Journal of Finance* 53, 1363–1387.

Chen, Ren-Raw, and Louis Scott, 1993, Maximum likelihood estimation for a multifactor equilibrium model of the term structure of interest rates, *Journal of Fixed Income* 3, 14–31.

Collin-Dufresne, Pierre, Robert S Goldstein, and Jean Helwege, 2010, Is credit event risk priced? modeling contagion via the updating of beliefs., Discussion paper, National Bureau of Economic Research.

Collin-Dufresne, Pierre, Robert S Goldstein, and J Spencer Martin, 2001, The determinants of credit spread changes, *The Journal of Finance* 56, 2177–2207.

De Jong, Frank, and Joost Driessen, 2012, Liquidity risk premia in corporate bond markets, *The Quarterly Journal of Finance* 2.

Dick-Nielsen, Jens, Jacob Gyntelberg, and Jesper Lund, 2013, From funding liquidity to market liquidity: Evidence from danish bond markets, *Copenhagen Business School Working Paper*.

Diebold, Francis X, Glenn D Rudebusch, and S Boragan Aruoba, 2006, The macroeconomy and the yield curve: A dynamic latent factor approach, *Journal of Econometrics* 131, 309–338.

Driessen, Joost, 2005, Is default event risk priced in corporate bonds?, *Review of Financial Studies* 18, 165–195.

Duffee, Gregory R, 1999, Estimating the price of default risk, *Review of Financial Studies* 12, 197–226.

Duffie, Darrell, Lasse Heje Pedersen, and Kenneth J Singleton, 2003, Modeling sovereign yield spreads: A case study of russian debt, *The Journal of Finance* 58, 119–159.

Duffie, Darrell, and Kenneth J Singleton, 1999, Modeling term structures of defaultable bonds, *Review of Financial Studies* 12, 687–720.

Elton, Edwin J, Martin J Gruber, Deepak Agrawal, and Christopher Mann, 2001, Explaining the rate spread on corporate bonds, *The Journal of Finance* 56, 247–277.

Eom, Young Ho, Jean Helwege, and Jing-zhi Huang, 2004, Structural models of corporate bond pricing: An empirical analysis, *Review of Financial studies* 17, 499–544.

Gallmeyer, Michael F, Burton Hollifield, and Stanley E Zin, 2005, Taylor rules, mccallum rules and the term structure of interest rates, *Journal of Monetary Economics* 52, 921–950.

Garleanu, Nicolae, and Lasse Heje Pedersen, 2011, Margin-based asset pricing and deviations from the law of one price, *Review of Financial Studies* 24, 1980–2022.

Giesecke, Kay, Francis A Longstaff, Stephen Schaefer, and Ilya Strebulaev, 2011, Corporate bond default risk: A 150-year perspective, *Journal of Financial Economics* 102, 233–250.

Gurkaynak, Refet S, Brian Sack, and Jonathan H Wright, 2007, The u.s. treasury yield curve: 1961 to the present, *Journal of Monetary Economics* 54, 2291–2304.

Hameed, Allaudeen, Wenjin Kang, and Shivesh Viswanathan, 2010, Stock market declines and liquidity, *The Journal of Finance* 65, 257–293.

Hamilton, James D, and Jing Cynthia Wu, 2012, Identification and estimation of gaussian affine term structure models, *Journal of Econometrics* 168, 315–331.

Hördahl, Peter, Oreste Tristani, and David Vestin, 2006, A joint econometric model of macroeconomic and term-structure dynamics, *Journal of Econometrics* 131, 405–444.

Joslin, Scott, Kenneth J Singleton, and Haoxiang Zhu, 2011, A new perspective on gaussian dynamic term structure models, *Review of Financial Studies* 24, 926–970.

Lin, Hai, Junbo Wang, and Chunchi Wu, 2011, Liquidity risk and expected corporate bond returns, *Journal of Financial Economics* 99, 628–650.

Longstaff, Francis A, Sanjay Mithal, and Eric Neis, 2005, Corporate yield spreads: Default risk or liquidity? new evidence from the credit default swap market, *The Journal of Finance* 60, 2213–2253.

Longstaff, Francis A, and Eduardo S Schwartz, 1995, A simple approach to valuing risky fixed and floating rate debt, *The Journal of Finance* 50, 789–819.

Luisi, Maurizio, and Jeffery D Amato, 2006, Macro factors in the term structure of credit spreads, .

Mueller, Philippe, 2008, Credit spreads and real activity, in *Workshop, Bank for International Settlements* vol. 29 p. 30.

Nelson, Charles R, and Andrew F Siegel, 1987, Parsimonious modeling of yield curves, *Journal of Business* pp. 473–489.

Wachter, Jessica A, 2006, A consumption-based model of the term structure of interest rates, *Journal of Financial Economics* 79, 365–399.

Wu, Liuren, and Frank Xiaoling Zhang, 2008, A no-arbitrage analysis of macroeconomic determinants of the credit spread term structure, *Management Science* 54, 1160–1175.

Table 1: Summary statistics of economic variables

Variables	Central moments				Autocorrelations		
	Mean	Stdev	Skew	Kurt	Lag 1	Lag 2	Lag 3
CPI	1.193	0.552	-0.263	4.515	0.950	0.862	0.775
PPI	1.056	1.112	-0.321	3.541	0.938	0.848	0.749
PCEde	0.967	0.449	0.002	3.936	0.963	0.899	0.828
GEMP	0.391	0.620	-1.648	6.104	0.971	0.945	0.908
GIP	0.840	1.831	-2.062	8.606	0.976	0.940	0.890
UE	6.064	1.595	0.887	2.840	0.995	0.989	0.980
3MTED	0.562	0.395	2.363	12.896	0.877	0.756	0.692
6MTED	0.582	0.358	1.789	7.971	0.928	0.859	0.806
3MCPMFFR	0.127	0.264	2.346	17.316	0.797	0.678	0.579
VXO	1.286	0.163	0.454	3.072	0.856	0.771	0.731
VIX	1.277	0.153	0.547	3.383	0.829	0.735	0.692

The table reports the summary statistics of economic variables. The inflation measures CPI, PPI, and PCEde are CPI inflation, PPI inflation, and PCE deflator respectively. The real activity measures GEMP, GIP, and UE are the growth rate of employment, the growth rate in industrial production, and the unemployment rate, respectively. The liquidity measures 3MTED, 6MTED, and 3MCPMFFR are 3-month LIBOR minus T-bill spread, 6-month LIBOR minus T-bill spread, and 3-month commercial paper minus federal funds rate, respectively. The volatility measures VIX and VXO are the VIX index and VXO index, respectively. The sample period is 1988:12 to 2013:05.

Table 2: Summary statistics of Treasury yields and credit spreads

	Central moments				Autocorrelations		
	Mean	Stdev	Skew	Kurt	Lag 1	Lag 2	Lag 3
Treasury Yields							
1 month	3.335	2.313	0.061	2.063	0.990	0.981	0.971
3 month	3.503	2.421	0.071	2.074	0.996	0.989	0.980
24 month	4.101	2.394	-0.079	2.081	0.993	0.984	0.973
60 month	4.735	2.128	-0.111	2.275	0.991	0.980	0.968
120 month	5.458	1.794	-0.048	2.453	0.989	0.977	0.967
A-rated Spreads							
24 month	1.220	1.204	4.127	22.261	0.961	0.903	0.843
60 month	1.129	0.867	3.222	16.064	0.962	0.906	0.844
120 month	1.074	0.770	2.885	13.673	0.965	0.920	0.871
BBB-rated Spreads							
24 month	1.871	1.431	2.974	14.088	0.965	0.913	0.855
60 month	1.750	1.100	3.152	16.337	0.964	0.905	0.832
120 month	1.642	0.937	2.750	13.763	0.968	0.914	0.851

The table reports the summary statistics of Treasury yields and credit spreads. The 1-, 3-, 24-, 60-, and 120-month Treasury yields are annual zero coupon bond yields. The 1-, 3-, 24-, 60-, and 120-months credit spreads under the A and BBB rating classes are calculated as the difference between the spot rate of the rating class and the Treasury yield with the same maturity. The sample period is 1988:12 to 2013:05.

Table 3: The dependence of the short rate on economic factors

	Inflation		Real Activity		Liquidity		Volatility	
Panel A:	Regressing short rate on economic factors (adjusted $R^2 = 0.52$)							
t	0.043	(9.600)	0.059	(9.974)	-0.018	(-2.944)	0.001	(0.177)
Panel B:	Regressing short rate on 12 lags of economic factors (adjusted $R^2 = 0.77$)							
t	0.023	(1.627)	0.098	(3.672)	0.028	(3.149)	0.012	(1.539)
t-1	-0.008	(-0.315)	0.022	(0.603)	0.006	(0.563)	0.003	(0.325)
t-2	0.005	(0.180)	-0.021	(-0.561)	-0.003	(-0.307)	0.011	(1.171)
t-3	0.003	(0.118)	0.002	(0.057)	-0.007	(-0.623)	0.006	(0.681)
t-4	-0.009	(-0.375)	-0.014	(-0.379)	0.003	(0.278)	-0.002	(-0.197)
t-5	0.004	(0.164)	-0.007	(-0.189)	0.002	(0.177)	-0.007	(-0.774)
t-6	0.008	(0.332)	0.000	(0.003)	-0.007	(-0.610)	-0.002	(-0.188)
t-7	0.000	(0.018)	0.010	(0.266)	-0.007	(-0.676)	-0.002	(-0.195)
t-8	0.011	(0.429)	0.002	(0.055)	-0.005	(-0.453)	-0.006	(-0.671)
t-9	0.000	(0.000)	-0.017	(-0.456)	-0.007	(-0.641)	0.004	(0.457)
t-10	-0.003	(-0.129)	-0.008	(-0.208)	-0.012	(-1.070)	-0.008	(-0.891)
t-11	0.014	(0.965)	0.043	(1.641)	-0.037	(-4.311)	-0.010	(-1.295)

The table reports the dependence of the short rate on economic factors. Panel A shows the results of regressing the 1-month yield on economic factors. Panel B shows the results of regressing the 1-month yield on 12 lags of economic factors. Standard errors are shown in parentheses.

Table 4: Mapping between structural and reduced-form parameters

VAR parameters	No. of elements	Σ_e 5	Σ_{mm} 10	Θ_{mm} 16	$\xi_{1m,1l}$ 16+9	α_1 7	Θ_{ll} 6	η_0 1	η_1 7	α_0 1	ξ_0 7
Ω_2^*	5	×									
Ω_m^*	10		×								
ϕ_{mm}^*	16			×							
ψ_{1m}^*	12			×	×	×			×		
ϕ_{21}^*	15				×	×	×		×		
Ω_1^*	6				×	×	×		×		
ϕ_{11}^*	9				×	×	×		×		
ϕ_{2m}^*	20			×	×	×	×		×		
ϕ_{1m}^*	12			×	×	×	×		×		
C_2^*	5	×	×	×	×	×	×	×	×	×	×
C_1^*	3		×	×	×	×	×	×	×	×	×

Table 5: Regressing Treasury yields on economic factors

Maturity	2 year	t-Statistic	5 year	t-Statistic	10 year	t-Statistic
Intercept	4.101	(40.733)	4.735	(50.020)	5.458	(65.536)
Inflation	0.599	(11.085)	0.571	(11.241)	0.511	(11.434)
Real activity	0.636	(8.806)	0.403	(5.926)	0.183	(3.066)
Adjusted R^2	0.480		0.418		0.366	
Intercept	4.101	(41.501)	4.735	(50.238)	5.458	(65.509)
Inflation	0.532	(9.254)	0.531	(9.692)	0.490	(10.117)
Real activity	0.655	(8.764)	0.408	(5.730)	0.188	(2.991)
Liquidity	-0.273	(-3.521)	-0.156	(-2.114)	-0.085	(-1.297)
Volatility	-0.068	(-0.807)	-0.055	(-0.680)	-0.023	(-0.323)
Adjusted R^2	0.499		0.423		0.366	

The table reports the parameter estimates from OLS regressions of Treasury yields on economic factors. The sample period is 1988:12 to 2013:05.

Table 6: Regressing A credit spreads on economic factors

Maturity	2 year	t-Statistic	5 year	t-Statistic	10 year	t-Statistic
Intercept	1.220	(23.438)	1.129	(27.203)	1.074	(28.453)
Inflation	0.029	(1.044)	0.077	(3.434)	0.029	(1.417)
Real activity	-0.569	(-15.237)	-0.358	(-12.003)	-0.298	(-11.016)
Adjusted R^2	0.450		0.327		0.294	
Intercept	1.220	(42.634)	1.129	(48.559)	1.074	(48.310)
Inflation	-0.061	(-3.681)	0.011	(0.814)	-0.024	(-1.862)
Real activity	-0.463	(-21.392)	-0.268	(-15.221)	-0.214	(-12.715)
Liquidity	-0.461	(-20.525)	-0.349	(-19.117)	-0.295	(-16.918)
Volatility	0.128	(5.250)	0.127	(6.427)	0.135	(7.145)
Adjusted R^2	0.834		0.789		0.755	

The table reports the parameter estimates from OLS regressions of A credit spreads on economic factors. The sample period is 1988:12 to 2013:05.

Table 7: Regressing BBB credit spreads on economic factors

Maturity	2 year	t-Statistic	5 year	t-Statistic	10 year	t-Statistic
Intercept	1.871	(35.221)	1.750	(37.138)	1.642	(38.393)
Inflation	0.002	(0.082)	0.034	(1.340)	0.010	(0.445)
Real activity	-0.765	(-20.057)	-0.527	(-15.588)	-0.409	(-13.311)
Adjusted R^2	0.595		0.460		0.388	
Intercept	1.871	(52.842)	1.750	(53.870)	1.642	(57.735)
Inflation	-0.039	(-1.878)	-0.009	(-0.487)	-0.022	(-1.349)
Real activity	-0.634	(-23.678)	-0.420	(-17.084)	-0.303	(-14.077)
Liquidity	-0.305	(-10.958)	-0.286	(-11.194)	-0.244	(-10.930)
Volatility	0.280	(9.267)	0.214	(7.715)	0.228	(9.384)
Adjusted R^2	0.820		0.743		0.729	

The table reports the parameter estimates from OLS regressions of Treasury yields on the BBB credit spreads. The sample period is 1988:12 to 2013:05.

Table 8: Model estimates

Θ_{1l}	0.9386	0	0	Θ_{1l}^Q	0.6660	0	0
	(76.934)				(34.426)		
	0	0.9934	0		-0.2778	0.9934	-0.0255
		(764.15)			(-10.289)	(764.15)	(-6.8919)
	0	0.0016	0.8262		-0.0298	0.0016	0.9793
		(2.2857)	(42.588)		(-2.0694)	(2.2857)	(489.65)
ξ_{0l}	-0.2893	0	0				
	(-9.3322)						
ξ_{1m}	0.6408	0.3906	0.1242	-0.4270	ξ_{1l}		
	0.1536	0.5547	0.1788	0.3376	0.2726	0	0
	-0.0160	-0.0208	0.1864	0.1446	0.2778	0	0.0255
	0.1826	0.1283	0.2158	-0.0809	0.0298	0	-0.1531
α_0	0.0027						
	(27.000)						
α_1	0.0004	0.0006	-0.0002	4.94E-06	1.46E-04	1.71E-04	6.51E-05
	(8.8300)	(10.187)	(-3.2733)	(0.0744)	(12.586)	(21.242)	(5.9724)
η_0^{BBB}	0.0044						
	(42.718)						
η_1^{BBB}	-0.0007	-0.0002	-0.0008	0.0002	3.72E-04	1.84E-04	-1.75E-04
	(-0.7928)	(-0.2488)	(-9.0293)	(3.3113)	(4.9799)	(16.140)	(-11.218)
η_0^A	0.0036						
	(32.143)						
η_1^A	-0.0011	-0.0008	-0.0020	-0.0008	-6.73E-05	1.48E-04	-9.08E-05
	(-1.4056)	(-2.162)	(-20.243)	(-5.2980)	(-2.6289)	(11.128)	(-11.792)
Σ_m	0.4305	0	0	0			
	0.0042	0.2272	0	0			
	0.0017	-0.0295	0.6905	0			
	-0.0453	0.0599	-0.2044	0.7256			

The table reports the model estimates and standard errors (in parentheses). The parameters are estimated with the minimum-chi-square methods. The sample period is 1988:12 to 2013:05.

Table 9: Forecast comparisons

| | RMSE criteria | | | MAD criteria | | |
	Two Factors	Four Factors	No-Arbitrage	Two Factors	Four Factors	No-Arbitrage
A24	0.0390	**0.0192**	0.0246	0.0268	**0.0126**	0.0146
A60	0.0380	0.0357	**0.0277**	0.0321	**0.0198**	0.0216
A120	0.0357	0.0367	**0.0312**	0.0302	**0.0191**	0.0223
BBB24	0.0442	0.0263	**0.0211**	0.0276	0.0215	**0.0117**
BBB60	0.0492	0.0411	**0.0410**	0.0430	**0.0307**	0.0317
BBB120	0.0462	0.0458	**0.0453**	0.0358	**0.0298**	0.0303

The table reports the comparisons of the out-of-sample forecasts for credit spreads. The forecasts are performed over the last 36 months of our sample, and the root mean square error (RMSE) and the mean absolute deviation (MAD) of the forecast versus the actual values are calculated. Lower RMSE and MAD denotes better forecasts. The best forecasts are highlighted in bold. We first estimate models on the in-sample, and update the estimations at each observation in the out-sample. Three models are considered: unrestricted VAR with two macro factors, unrestricted VAR with two macro factors, and two financial market factors and VAR with four factors and cross-equation restrictions.
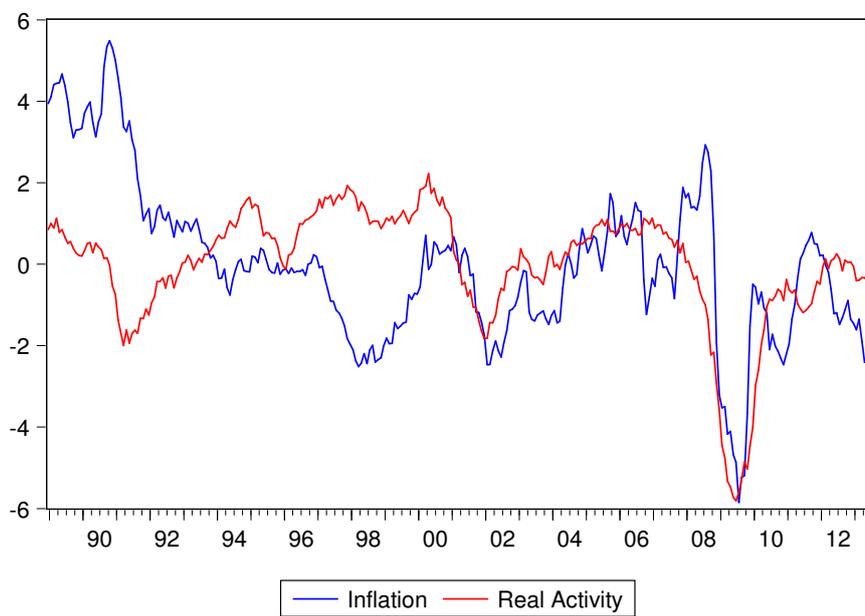
Figure 1: Macro factors: inflation and real activity. The figure illustrates the two macro factors representing inflation (blue) and real activity(red). The sample period is 1988:12 to 2013:05.
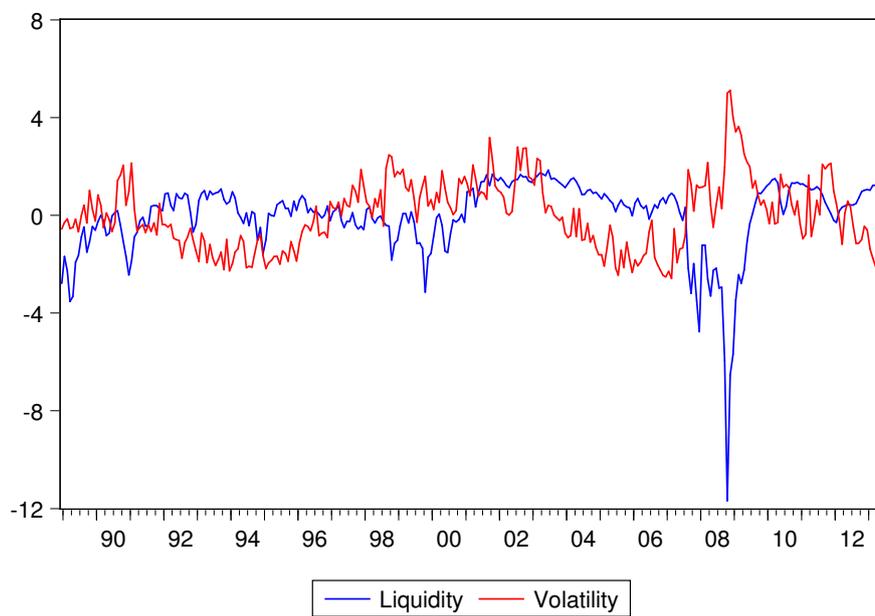
Figure 2: Financial market factors: liquidity and volatility. The figure illustrates the two financial market factors representing liquidity (blue) and volatility (red). The sample period is 1988:12 to 2013:05.
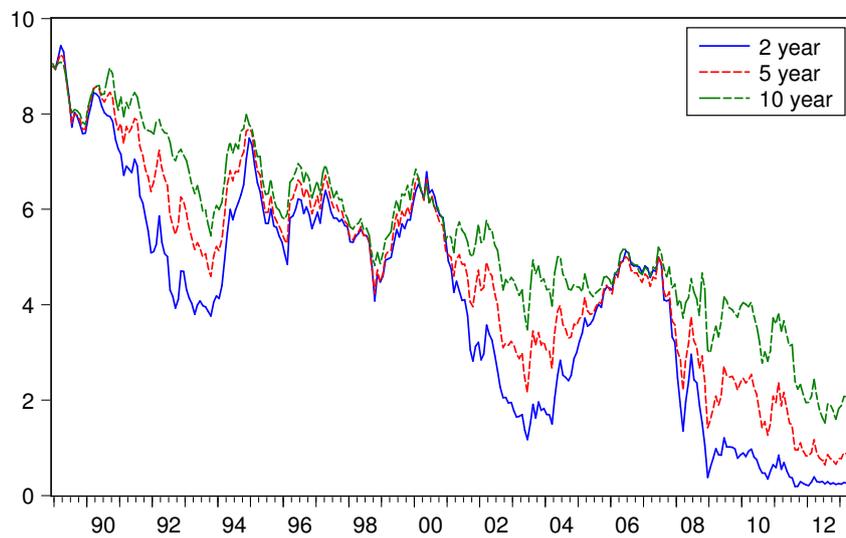
Figure 3: Treasury yields. This figure illustrates (annualized) monthly zero coupon bond yields of maturity 2 years (blue), 5 years (red), and 10 years (green). The sample period is 1988:12 to 2013:05.
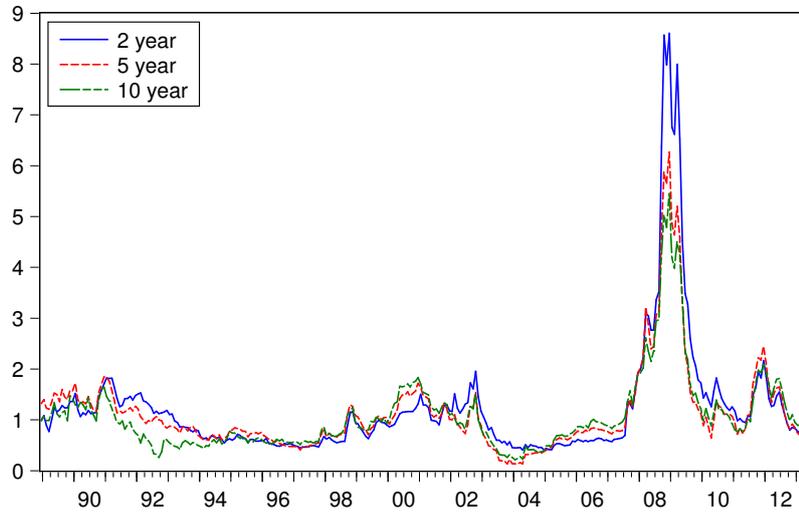
(a) A credit spreads
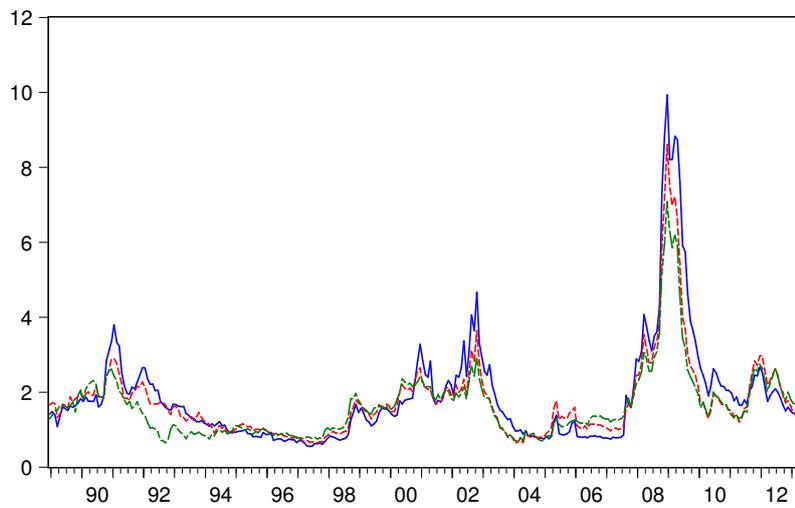
(b) BBB credit spreads

Figure 4: Credit spreads: A and BBB rating classes. Panel (a) illustrates the credit spreads at maturity 2 years, 5 years, and 10 years under A rating class. Panel (b) illustrates the credit spreads at maturity 2 years, 5 years, and 10 years under the BBB rating class.
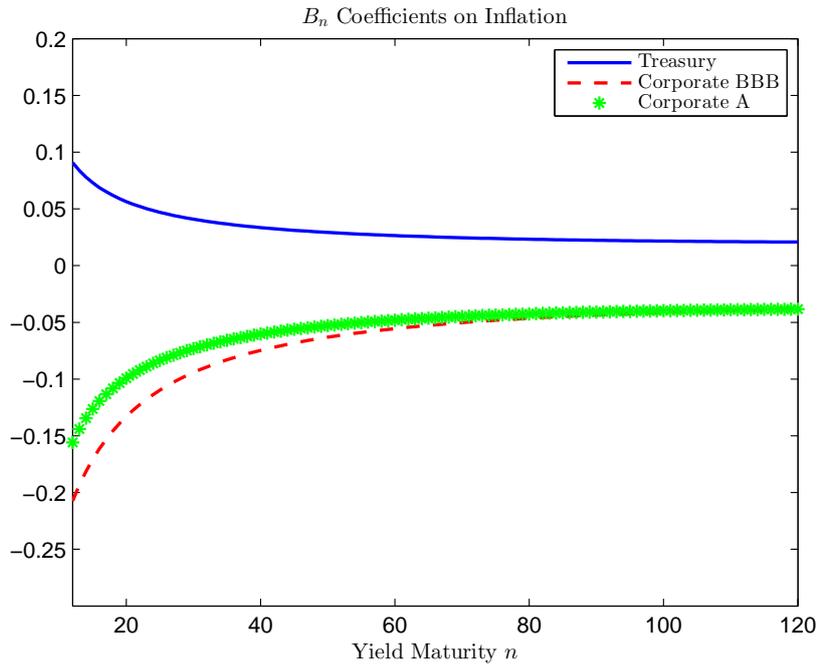
(a) Inflation factor
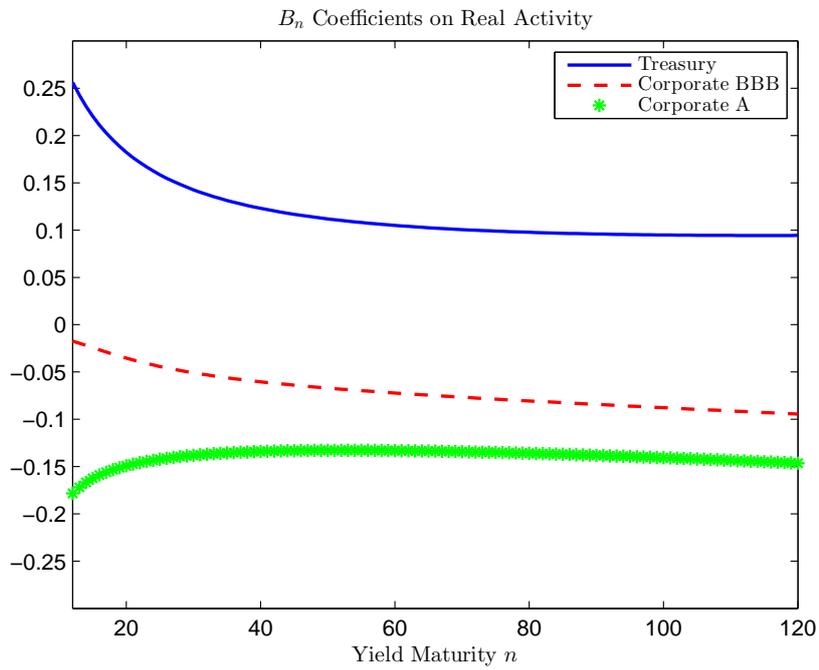
(b) Real activity factor

Figure 5: Loadings on macro factors. The figure shows how Treasury yields and corporate yields change respond to a one standard deviation change in macro factors. Panel (a) shows the loadings on the inflation factor and Panel (b) shows the loadings on real activity factor. Yields maturity is expressed in months.

56
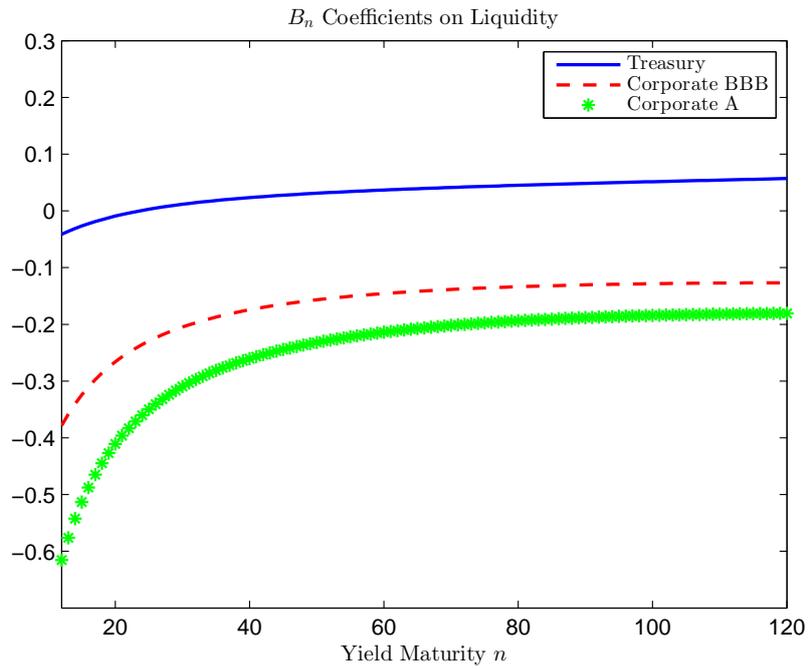
(a) Liquidity factor
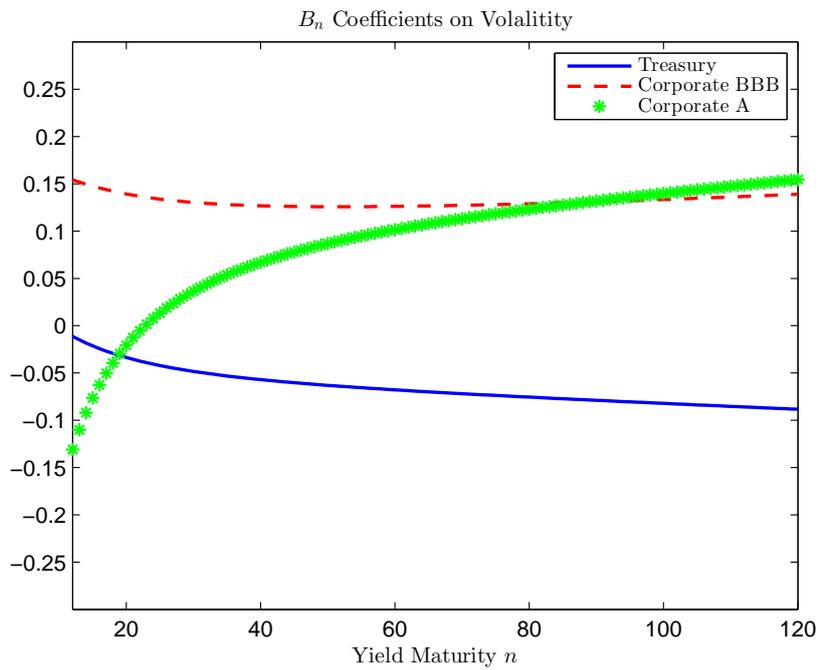
(b) Volatility factor

Figure 6: Loadings on financial market factors. The figure shows how Treasury yields and corporate yields respond to a one standard deviation change in financial market factors. Panel (a) shows the loadings on liquidity factor and panel (b) shows the loadings on the volatility factor. Yields maturity is expressed in months.
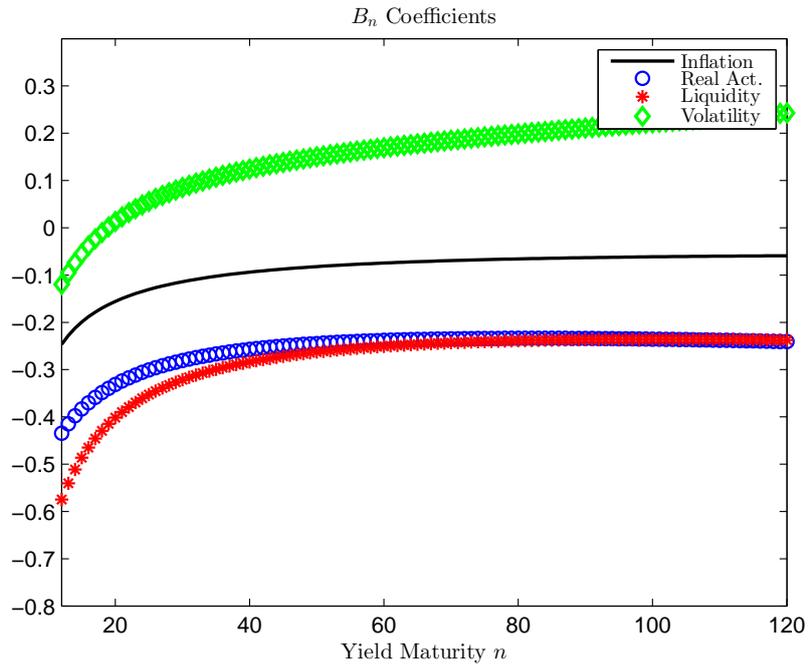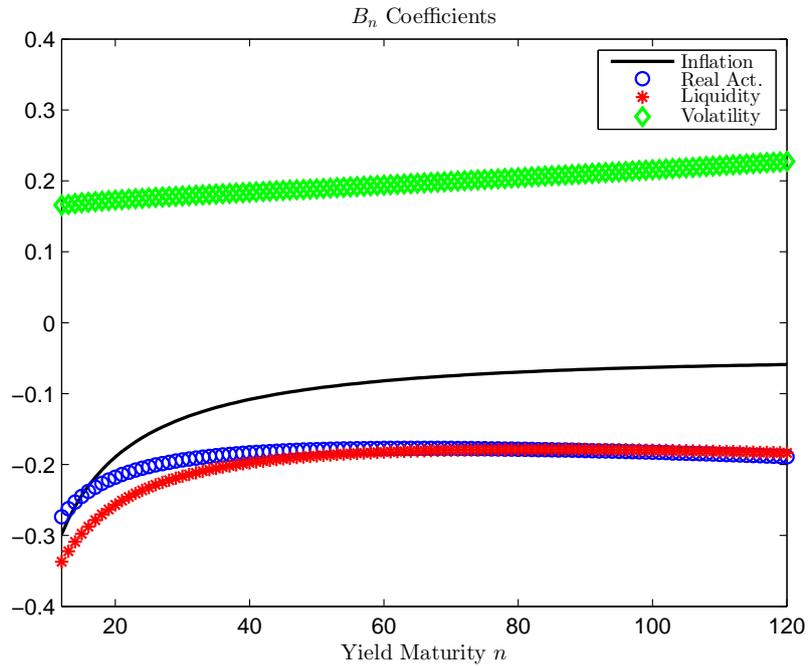
(a) A credit spreads

(b) BBB credit spreads

Figure 7: Loadings of credit spreads on economic factors. The figure shows how credit spreads respond to a one standard deviation change in economic factors. Panel (a) shows the loadings of the A credit spreads, and panel (b) shows the loadings of the BBB credit spreads. Yields maturity is expressed in months.
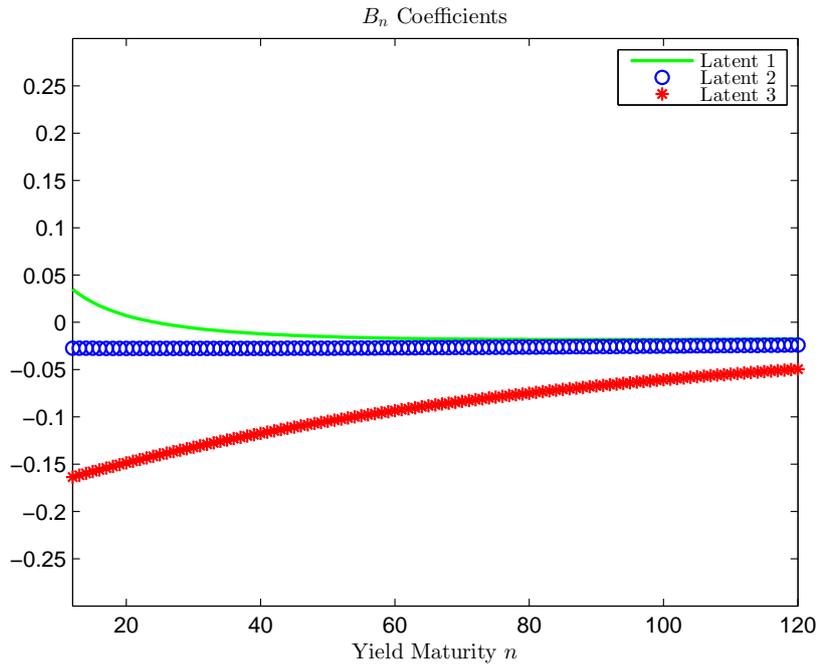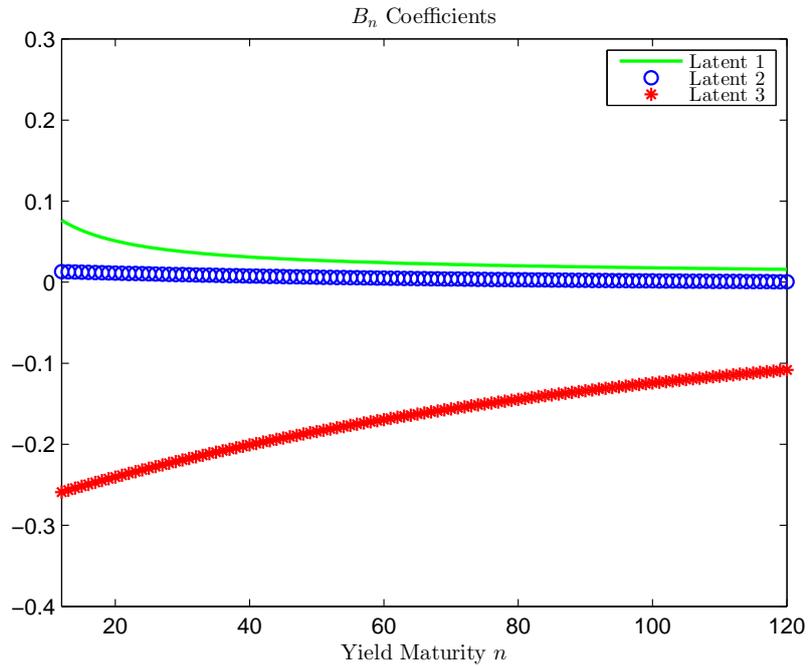
(a) A credit spreads

(b) BBB credit spreads

Figure 8: Loadings of credit spreads on latent factors. The figure shows how credit spreads respond to a one standard deviation change in latent factors. Panel (a) shows the loadings of the A credit spreads and panel (b) shows the loadings of the BBB credit spreads. Yields maturity is expressed in months.
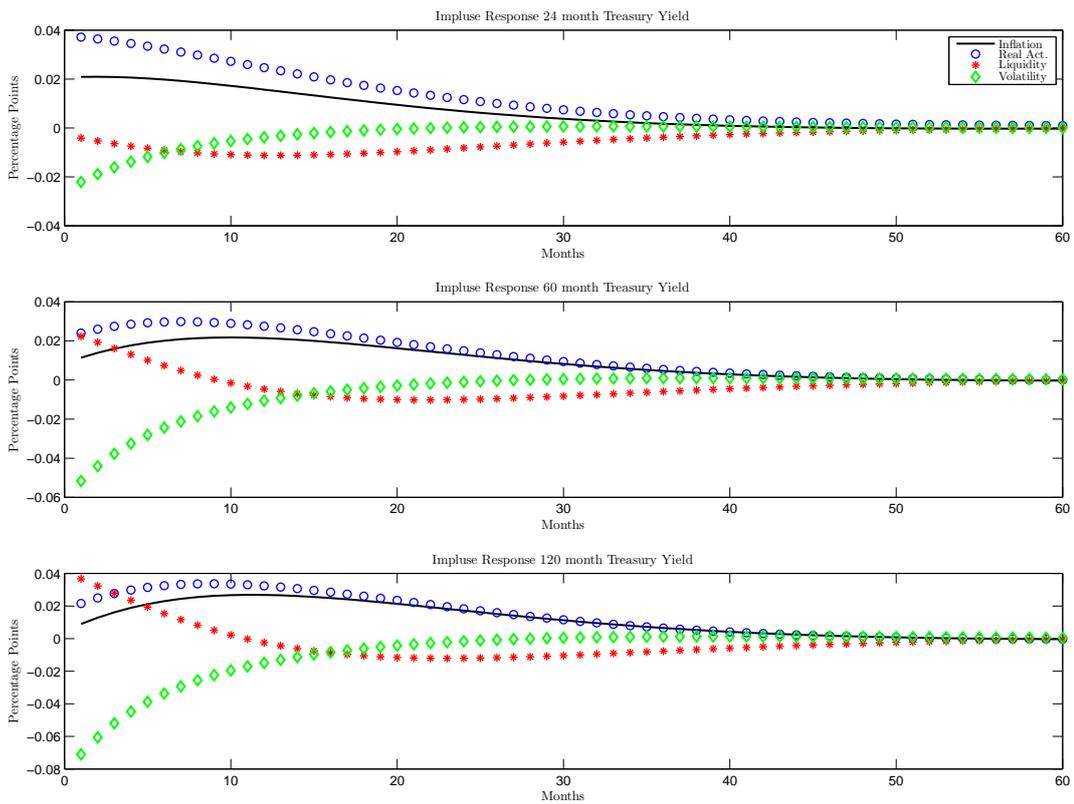
Figure 9: Impulse responses functions for Treasury yields. The figure shows impulse responses for 24-months (top), 60-months (middle), 120-months (bottom) Treasury yields. Only the impulse responses from the economic factors are shown. All impulse responses are from a one standard deviation shock.
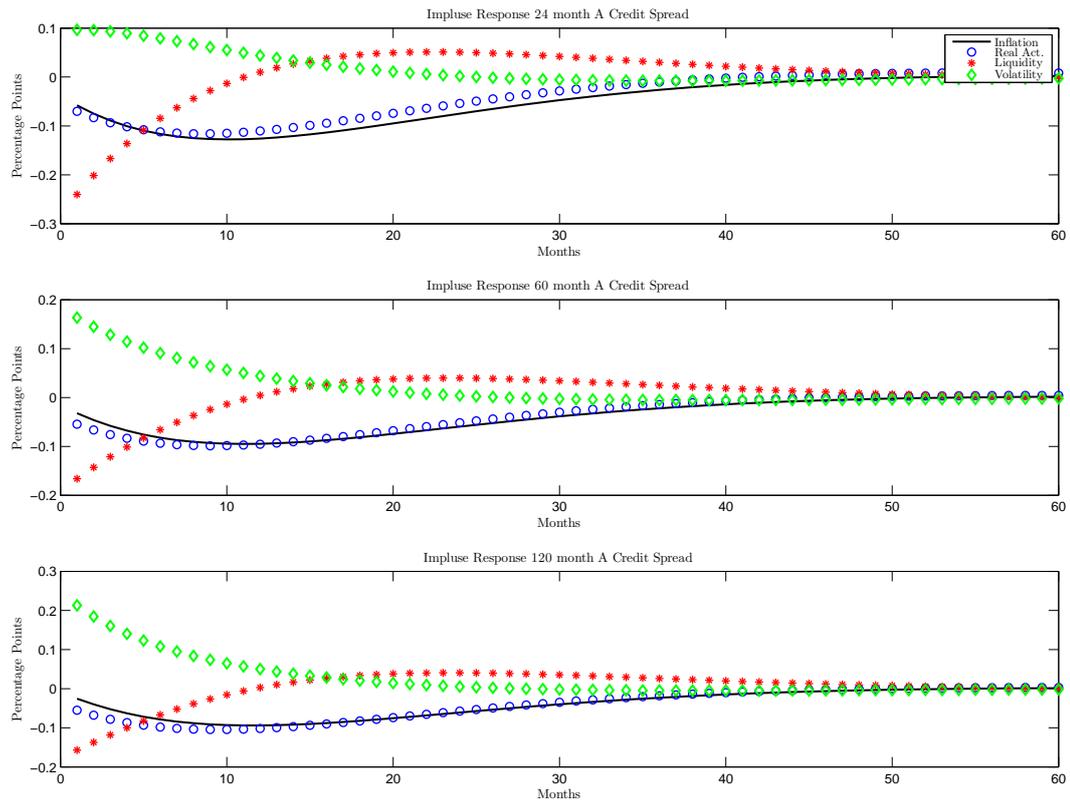
Figure 10: Impulse responses functions for A credit spreads. The figure shows impulse responses for 24-months (top), 60-months (middle), 120-months (bottom) A-rated bond yields. Only the impulse responses from the economic factors are shown. All impulse responses are from a one standard deviation shock.
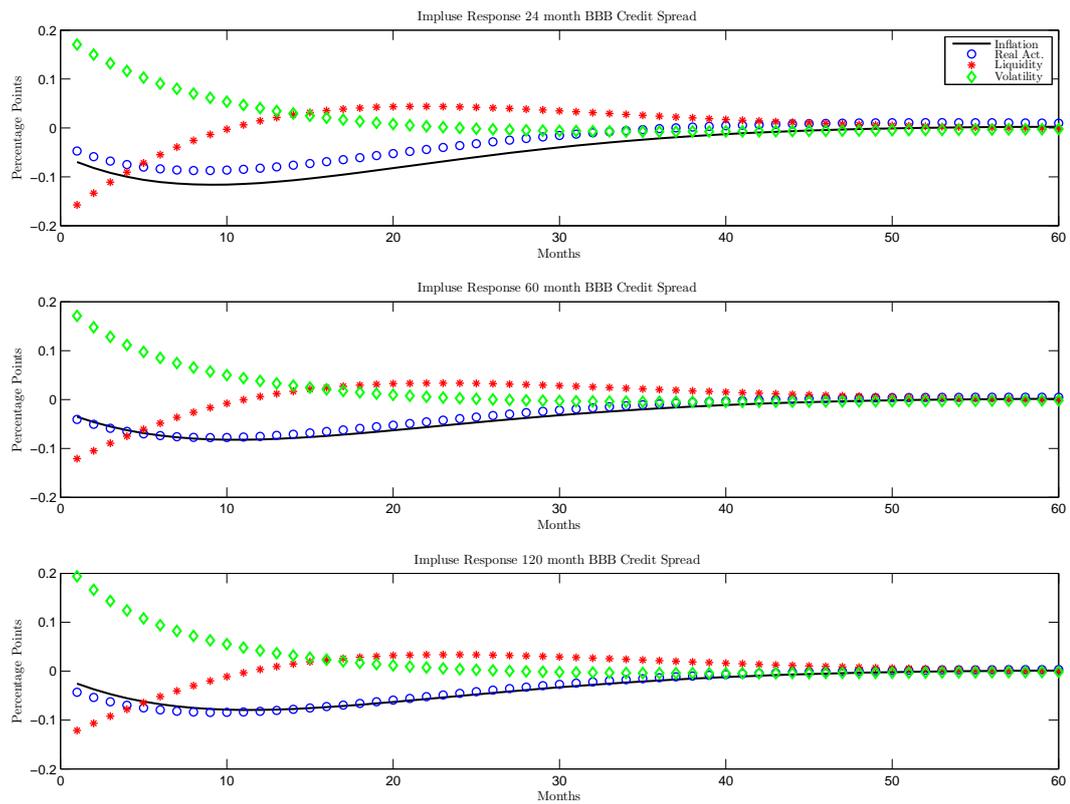
Figure 11: Impulse responses functions for BBB credit spreads. The figure shows impulse responses for 24-months (top), 60-months (middle), 120-months (bottom) BBB-rated bond yields. Only the impulse responses from the economic factors are shown. All impulse responses are from a one standard deviation shock.